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NONLINEAR CONTROL FOR HIGH-ANGLE-OF-ATTACK AIRCRAFT FLIGHT USING THE STATE-DEPENDENT RICCATI EQUATION

Introduction

Flight control systems for aerospace vehicles present significant challenges for nonlinear flight regimes such as high-angle-of-attack flight. In such flight regimes linear controllers may not execute a desired performance. Therefore, nonlinearities of the vehicle dynamics must be taken into account by a control algorithm.

State-dependent Riccati equation (SDRE) methodis a heuristic technique that was originally proposed by Pearson [1] and independently studied by Cloutier et al [2]-[6]. In SDRE control a nonlinear system is parameterized to have a linear-like structure. The optimal control is obtained by solving a SDRE at every point on the trajectory.SDRE algorithm captures the nonlinearities of the system by converting it to a quasi-linear structure using state-dependent coefficient (SDC) matrices. This enables the re-computing of the controller gains in real time by minimizing a quasi-quadratic cost function. An algebraic Riccati equation (ARE) using SDC matrices is solved on-line to obtain the feedback gain. The non-uniqueness of the parameterization creates additional degrees of freedom, which may be used to enhance controller performance. It is important to note that methods using the SDRE can be applied to minimum as well as a non-minimum phase nonlinear system. Furthermore, the weight may be adaptively changed to avoid actuator saturation problems.

SDRE approach is applied to a number of control problems in aerospace applications, such as missile control [6], [7], control for VTOL vehicles [8], [9], and quadrotors [10]. Another wide area of SDRE application is a spacecraft attitude control [11]–[14]. However, the utilization of SDRE control to fixed-wing aircraft that operate in nonlinear flight regimes is not explored.

This paper focuses on the application of SDRE method for the flight control of a fixed-wing unmanned aerial vehicle. The control algorithm represents a tracking controller and consists of two cascaded control loops. The outer loop addresses control of the attitude and altitude of the aircraft, and the inner loop is used to control rotational and translational velocities. In addition, a nonlinear compensator is implemented to account for the mismatch between the full vehicle dynamics and its SDC parameterization, that occurs the inner loop. Performance of the SDRE controller is demonstrated using a nonlinear simulation model of the aircraft for a high angle of attack maneuver.

Problem Formulation

The task of this paper is to develop a nonlinear flight control system for a fixed-wing aircraft based on SDRE method. The controller dual-loop structure involves development of the SDC models of the aircraft dynamics for each loop, and also a nonlinear compensator that cancels miss-match between the actual and modeled dynamics.

SDRE Control and SDC Parameterization

SDRE control method involves factorization of the nonlinear dynamics

$$\dot{x}(t) = f(x(t), u(t)),\ x(0) = x_0, \tag{1}$$

where $x \in \mathbf{R}^n$ is the state vector, $u \in \mathbf{R}^m$ is the input vector, function $f:\ \mathbf{R}^n \to \mathbf{R}^n$. SDC parameterization yields the linear-like structure with SDC matrices given by

$$\dot{x}(t) = A(x(t), u(t))\, x(t) + B(x(t), u(t)) u(t),\ x(0) = x_0, \tag{2}$$

where $A:\ \mathbf{R}^n \to \mathbf{R}^{n\times n}$ and $B:\ \mathbf{R}^n \to \mathbf{R}^{n\times m}$. It should be notes that SDC dynamics matrixA in (2) is not unique when $n > 1$, [15].

The performance cost function to be minimized is defined as

$$J(x_0, u) = \frac{1}{2}\int_0^\infty \left\{ x^{\mathrm{T}}(t) R_1(x(t)) x(t) + u^{\mathrm{T}}(t) R_2(x(t)) u(t) \right\} \mathrm{d}t, \tag{3}$$

where $R_1(x(t)) \in \mathbf{R}^{n\times n}$ is positive semidefinite, and $R_2(x(t)) \in \mathbf{R}^{n\times n}$ is positive definite. SDRE method requires that the pair $A(x(t), u(t)), B(x(t), u(t))$ must be pointwise stabilizable, and full state vector measurements must be available for feedback.

Let $A(x) \triangleq A(x(t), u(t)), B(x) \triangleq B(x(t), u(t))$. The state feedback control law is given by

$$u(t) = -K(x) x(t) = -R_2^{-1}(x) B^{\mathrm{T}}(x) P(x) x(t), \tag{4}$$

where $P(x) \triangleq P(x(t))$ is a solution of the state-dependent algebraic Riccati equation, [16]

$$A^{\mathrm{T}}(x) P(x) + P(x) A(x) - P(x) B(x) R_2^{-1}(x) B^{\mathrm{T}}(x) P(x) + R_1(x) = 0. \tag{5}$$

It is important to emphasize that SDRE method is heuristic since the control law is suboptimal with respect to the performance index (3) and may not be stabilizing. Some conditions for stability of SDRE method for high-order systems are given in Ref. [4].

Tracking Controller and Compensator

In tracking (trajectory following) systems, it is required that the outputs precisely follow desired trajectories in some optimal sense. Optimality is reached by minimization of a given cost function. Naidu [17] and Anderson [18] show a linear quadratic tracking (LQT) controller that aims to maintain the output as close as possible to the desired reference input with minimum control energy for an observable linear time-varying system.

Consider a nonlinear system in the SDC form

$$\begin{aligned} \dot{x}(t) &= A(x)x(t) + B(x)u(t) + f(x), \; x(0) = x_0, \\ y(t) &= C(x)x(t), \end{aligned} \tag{6}$$

where $f(x)$ represents a mismatch that appears as a result of the SDC factorization of the nonlinear system, provided that $f(x)$ is slowly varying and bounded. It is desired to control system (6) such that the desired output $y(t)$ tracks the reference input $z(t)$.

Reference [19] provides derivations of the infinite horizon tracking controller and a compensator, minimizing a performance index

$$\lim_{t_{\mathrm{f}}\to\infty} J(x_0, u) = \lim_{t_{\mathrm{f}}\to\infty} \frac{1}{2}\int_0^{\infty} \left\{ e^{\mathrm{T}}(t)Q(t)e(t) + u^{\mathrm{T}}(t)R(t)u(t) \right\} \mathrm{d}t, \tag{7}$$

where $e(t) = z(t) - y(t)$ is the tracking error.

The control law for system (6) can be written in the form

$$u(t) = K(x)x(t) + K_{\mathrm{z}}(x)z(t) + K_{\mathrm{f}}(x)f(t). \tag{8}$$

Corresponding controller gains are defined as

$$\begin{aligned} K(x) &= -R^{-1}(x)B^{\mathrm{T}}(x)P(x), \\ K_{\mathrm{z}}(x) &= -R^{-1}(x)B^{\mathrm{T}}(x)[P(x)E(x) - A^{\mathrm{T}}(x)]^{-1}W(x), \\ K_{\mathrm{f}}(x) &= -R^{-1}(x)B^{\mathrm{T}}(x)[P(x)E(x) - A^{\mathrm{T}}(x)]^{-1}P(x), \end{aligned} \tag{9}$$

where $P(x)$ is a solution of the state-dependent algebraic Riccati equation

$$A^{\mathrm{T}}(x)P(x) + P(x)A(x) - P(x)B(x)R^{-1}(x)B^{\mathrm{T}}(x)P(x) + C^{\mathrm{T}}(x)Q(x)C(x) = 0, \tag{10}$$

and

$$\begin{aligned} E(x) &= B(x)R^{-1}(x)B^{\mathrm{T}}(x), \\ W(x) &= C^{\mathrm{T}}(x)Q(x). \end{aligned} \tag{11}$$

The gain $K_{\mathrm{f}}(x)$ represents a compensator and is used to cancel the mismatch term $f(x)$ in the SDC model (6).

Controller Structure and Extended Parameterization

The proposed flight control system consists of two concentric loops and its block diagram is shown in Fig. 1. For each control loop a separate SDRE tracking controller is implemented. The main advantage of this two-loop architecture is the reduction in the dimensions of state vectors, and computational cost associated with the calculation of the feedback gains.

The outer loop is used to control the angular position of an aircraft and its altitude. Inner loop controls the translational and rotational velocities of the vehicle. Control inputs include using of the elevator, ailerons, rudder and throttle. A reference input computing block contains a simple navigation algorithm that generates consistent commands to the outer loop.

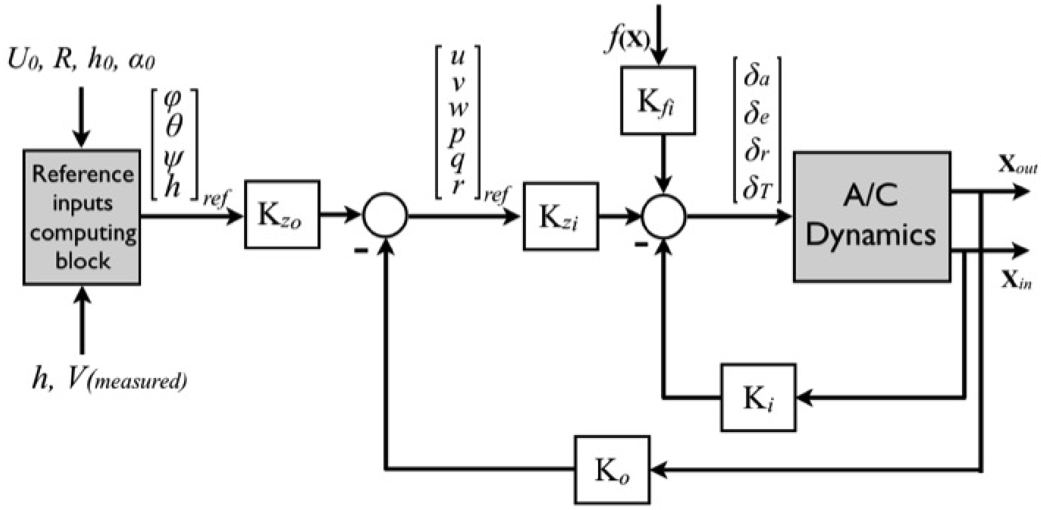

Fig. 1. Control System Block Diagram

The 6 degrees-of-freedom equations of motion of an aircraft written in the body-fixed coordinate system are used to obtain the SDC models for the inner loop, [20]. Kinematic equations are utilized to relate the body fixed measurements to the altitude and attitude.

The outer loop state and control vectors are defined as follows:

$$x_{\mathrm{out}} = [\phi, \theta, \psi, h]^{\mathrm{T}}, u_{\mathrm{out}} = [u, v, w, p, q, r]^{\mathrm{T}},$$

where ϕ, θ, ψ are the Euler angles, h is the altitude; u, v, w are the components of the translational velocity, written in body axis; p, q, r are the components of the rotational velocity, written in body axis. A possible set of SDC matrices for the outer loop dynamics can be written as

$$A_{\mathrm{out}}(x_{\mathrm{out}}) = [0],$$

$$B_{\mathrm{out}}(x_{\mathrm{out}}) = \begin{bmatrix} 0 & 0 & 0 & 1 & \tan\theta\sin\phi & \tan\theta\cos\phi \\ 0 & 0 & 0 & 0 & \cos\phi & -\sin\phi \\ \sin\theta & \cos\theta\sin\phi & -\cos\theta & 0 & 0 & 0 \end{bmatrix},$$

$$f_{\mathrm{out}}(x_{\mathrm{out}}) = [0].$$

The outer loop state and control vectors are defined as follows:

$$x_{\text{in}} = \left[u, v, w, p, q, r\right]^{\mathrm{T}}, u_{\text{in}} = \left[\delta_{\mathrm{a}}, \delta_{\mathrm{e}}, \delta_{\mathrm{r}}, \delta_{\mathrm{T}}\right]^{\mathrm{T}},$$

where $\delta_{\mathrm{a}}, \delta_{\mathrm{e}}, \delta_{\mathrm{r}}, \delta_{\mathrm{T}}$ present aileron, elevator, rudder and throttle inputs, respectively.

A possible choice of the state-dependent dynamics and input matrices for the inner loop dynamics model can be obtained in thefollowing form

$$A_{\text{in}}(x_{\text{in}}) = \begin{bmatrix} A_{11}(x_{\text{in}}) & A_{12}(x_{\text{in}}) \\ A_{21}(x_{\text{in}}) & A_{22}(x_{\text{in}}) \end{bmatrix},$$

where

$$A_{11}(x_{\text{in}}) = \begin{bmatrix} -\dfrac{\rho VS}{2m}(C_{\mathrm{D}_0} + C_{\mathrm{D}_\alpha}\alpha) & 0 & \dfrac{\bar{q}S}{mu}(C_{\mathrm{L}_\alpha} + C_{\mathrm{L}_0}) \\ 0 & \dfrac{\bar{q}SC_{\mathrm{Y}_\beta}}{mu} & 0 \\ -\dfrac{\rho VS}{2m}(C_{\mathrm{D}_0}\alpha + C_{\mathrm{L}_0}) & 0 & -\dfrac{\bar{q}S}{mu}(C_{\mathrm{D}_\alpha}\alpha + C_{\mathrm{L}_\alpha}) \end{bmatrix},$$

$$A_{12}(x_{\text{in}}) = \begin{bmatrix} 0 & \dfrac{\bar{q}S\bar{c}^2}{2mV}(C_{\mathrm{L}_q} + C_{\mathrm{L}_{\dot{\alpha}}})\alpha - w & v \\ \dfrac{\bar{q}Sb^2}{2mV}C_{\mathrm{Y}_p} + w & \dfrac{\bar{q}SC_{\mathrm{Y}_\beta}}{mu} & \dfrac{\bar{q}Sb^2}{2mV}C_{\mathrm{Y}_r} - u \\ -v & 0 & 0 \end{bmatrix},$$

$$A_{21}(x_{\text{in}}) = \begin{bmatrix} 0 & \dfrac{\bar{q}Sb}{u}(c_3 C_{\mathrm{l}_\beta} + c_4 C_{\mathrm{n}_\beta}) & 0 \\ \dfrac{\rho VS\bar{c}}{2I_{\mathrm{yy}}}C_{\mathrm{m}_0} & 0 & \dfrac{\bar{q}VS\bar{c}}{I_{\mathrm{yy}}u}C_{\mathrm{m}_\alpha} \\ 0 & \dfrac{\bar{q}Sb}{u}(c_4 C_{\mathrm{l}_\beta} + c_5 C_{\mathrm{n}_\beta}) & 0 \end{bmatrix},$$

$$A_{22}(x_{\text{in}}) = \begin{bmatrix} \dfrac{\bar{q}Sb^2}{2V}(c_3 C_{\mathrm{l}_p} + c_4 C_{\mathrm{n}_p}) & 0 & \dfrac{\bar{q}Sb^2}{2V}(c_3 C_{\mathrm{l}_r} + c_4 C_{\mathrm{n}_r}) + c_1 q \\ 0 & \dfrac{\bar{q}S\bar{c}^2}{2V}c_6(C_{\mathrm{m}_q} + C_{\mathrm{m}_{\dot{\alpha}}}) & 0 \\ \dfrac{\bar{q}Sb^2}{2V}(c_4 C_{\mathrm{l}_p} + c_5 C_{\mathrm{n}_p}) + c_8 q & 0 & \dfrac{\bar{q}Sb^2}{2V}(c_4 C_{\mathrm{l}_r} + c_5 C_{\mathrm{n}_r}) - c_2 q \end{bmatrix},$$

$$B_{\text{in}}(x_{\text{in}}) = \begin{bmatrix} B_1(x_{\text{in}}) & B_2(x_{\text{in}}) & B_2(x_{\text{in}}) \end{bmatrix},$$

where

$$B_1(x_{\text{in}}) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \overline{q}Sb(c_3 C_{\text{l}_{\delta a}} + c_4 C_{\text{n}_{\delta a}}) \\ 0 \\ \overline{q}Sb(c_4 C_{\text{l}_{\delta a}} + c_5 C_{\text{n}_{\delta a}}) \end{bmatrix}, B_2(x_{\text{in}}) = \begin{bmatrix} \frac{\overline{q}S}{m}(C_{\text{L}_{\delta e}}\alpha - C_{\text{D}_{\delta e}}) \\ 0 \\ -\frac{\overline{q}S}{m}(C_{\text{L}_{\delta e}} + C_{\text{D}_{\delta e}}\alpha) \\ 0 \\ \overline{q}S\overline{c}c_6 C_{\text{m}_{\delta e}} \\ 0 \end{bmatrix},$$

$$B_3(x_{\text{in}}) = \begin{bmatrix} 0 \\ \frac{\overline{q}S}{m} C_{\text{Y}_{\delta r}} \\ 0 \\ \overline{q}Sb(c_3 C_{\text{l}_{\delta r}} + c_4 C_{\text{n}_{\delta r}}) \\ 0 \\ \overline{q}Sb(c_4 C_{\text{l}_{\delta r}} + c_5 C_{\text{n}_{\delta r}}) \end{bmatrix}, B_4(x_{\text{in}}) = \begin{bmatrix} \frac{C_{\text{T}}}{m} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix},$$

and

$$c_1 = \frac{(I_{\text{yy}} - I_{\text{zz}})I_{\text{zz}} - I_{\text{xz}}^2}{I_{\text{xx}}I_{\text{zz}} - I_{\text{xz}}^2}; \quad c_2 = \frac{(I_{\text{xx}} - I_{\text{yy}} + I_{\text{zz}})I_{\text{xz}}}{I_{\text{xx}}I_{\text{zz}} - I_{\text{xz}}^2}; \quad c_3 = \frac{I_{\text{zz}}}{I_{\text{xx}}I_{\text{zz}} - I_{\text{xz}}^2};$$

$$c_4 = \frac{I_{\text{xz}}}{I_{\text{xx}}I_{\text{zz}} - I_{\text{xz}}^2}; \quad c_5 = \frac{I_{\text{xx}}}{I_{\text{xx}}I_{\text{zz}} - I_{\text{xz}}^2} \quad c_6 = \frac{1}{I_{\text{yy}}}.$$

The mismatch between the original dynamics and the SDC parameterization includes terms that appeardue to the gravitational acceleration is modeled as a slowly varying external input

$$f(x_{\text{in}}) = \begin{bmatrix} -g\sin\theta \\ g\cos\theta\sin\phi \\ g\cos\theta\cos\phi \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

Simulation Results

A nonlinear simulation model of a fixed-wing unmanned aircraft is used to verify the performance of the designed SDRE controller. The aircraft has a mass of 105 kg, wing span 4.3 m, and chord length 0.53 m. The aerodynamic coefficients are in the form of look-up tables and include the nonlinearities such as drop in the aerodynamic lift coefficient and increase in aerodynamic drag coefficient at high values of the angle of attack. The actuators are modeled as first-order servos.

Selection of the weighting matrices Q and R is a crucial step in designing a SDRE controller. For the purposes of this work, matrices Q and R are chosen to be constant diagonal matrices with the following diagonal entries

$$\begin{aligned}
Q_{\text{out}} &= \text{diag}[10^2, 10^2, 10^2, 0.6],\\
R_{\text{out}} &= \text{diag}[1, 1, 3, 1, 1, 1],\\
Q_{\text{in}} &= \text{diag}[10^3, 10^3, 10^3, 10^4, 2\times 10^5, 10^4],\\
R_{\text{in}} &= \text{diag}[1, 1, 1, 10^{-3}].
\end{aligned}$$

To demonstrate effectiveness of the designed flight control system in flight regime that covers the nonlinear regions of the aerodynamic lift coefficient curve, a level flight at a high angle of attack is simulated. Commanding a high pitch angle and holding the altitude constant allows achieving this flight regime. The reference pitch attitude is set to 18 deg and a required altitude is 1000 m. Roll and yaw angles are commands are zero. The update frequency for the controllers' gains is 2 Hz.

The aerodynamic lift coefficient versus angle of attack plot is shown in Fig. 2, from which it may be observed that the stall value is around 10 deg.

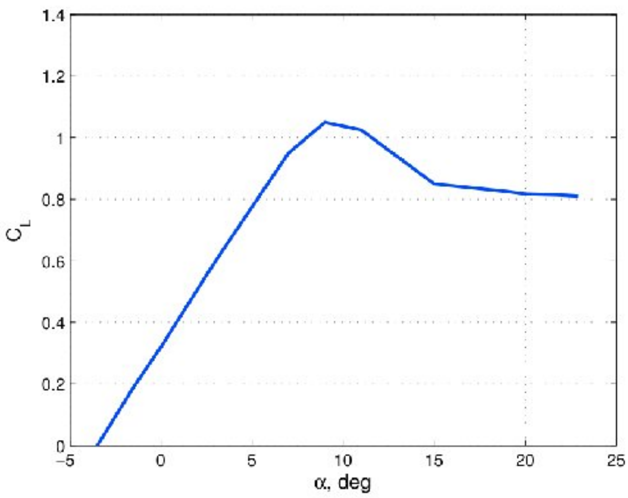

Fig. 2. Lift Coefficient vs Angle of Attack

The angle of attack response is given in Fig. 3, from which it can be observed that the aircraft operates at the high angle of attack flight regime, which corresponds to the nonlinear region in the aerodynamic lift curve. Pitch angle and altitude responses in Fig. 4 show that a level flight condition is achieved despite of a small steady state altitude error.

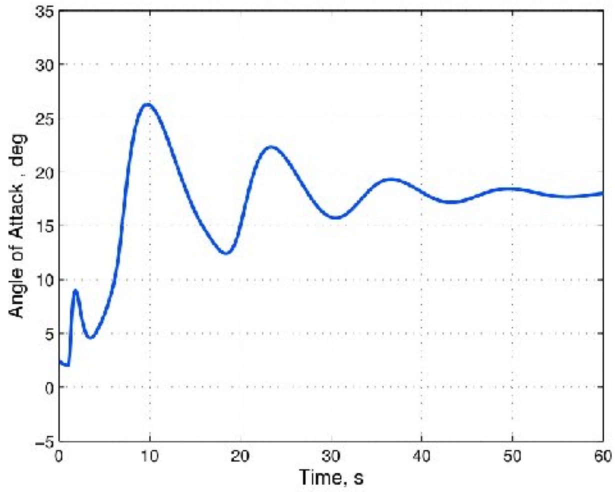

Fig. 3. Angle of Attack

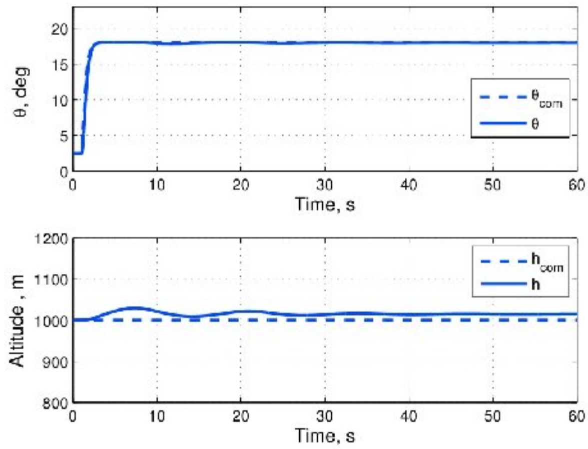

Fig. 4. Pitch Angle and Altitude

Responses of the inner loop states that include linear and rotational velocity components are shown in Fig. 5. Thrust and elevator responses are presented in Fig. 6.

Time histories of inner and outer loop controller gains are given in Fig. 7 - 11, and show that controller's gains are re-adjusted according to the flight regime, ensuring sufficient tracking performance of the controller.

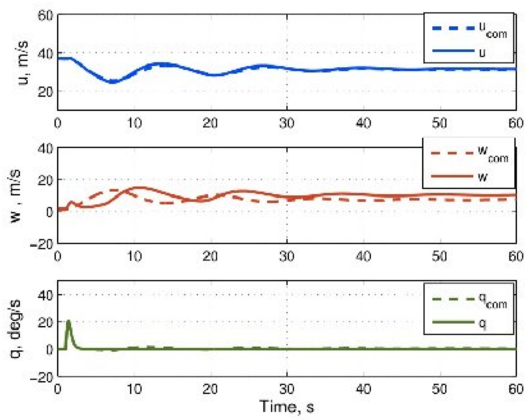

Fig. 5. Linear and Angular Velocities Components

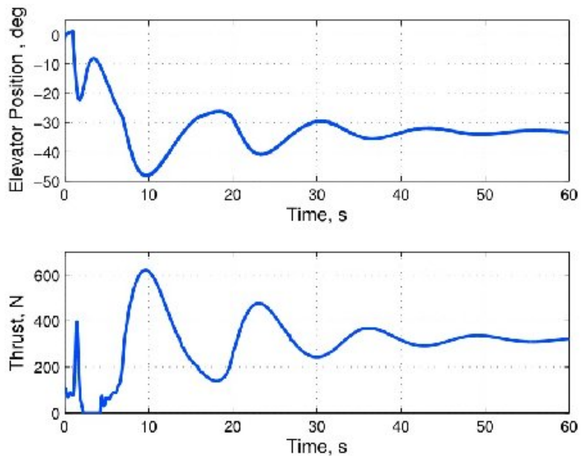

Fig. 6. Elevator Position and Thrust

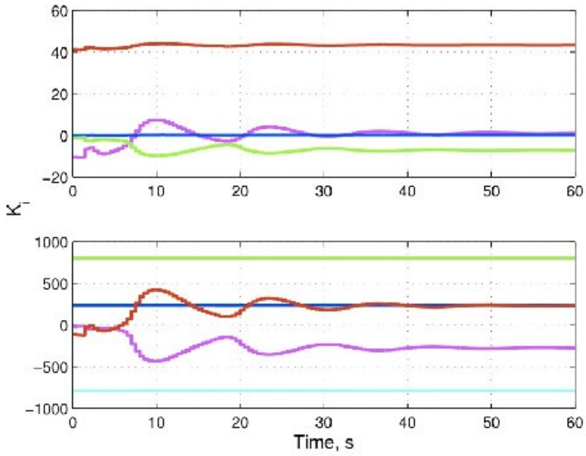

Fig. 7. Inner Loop Controller Gain K

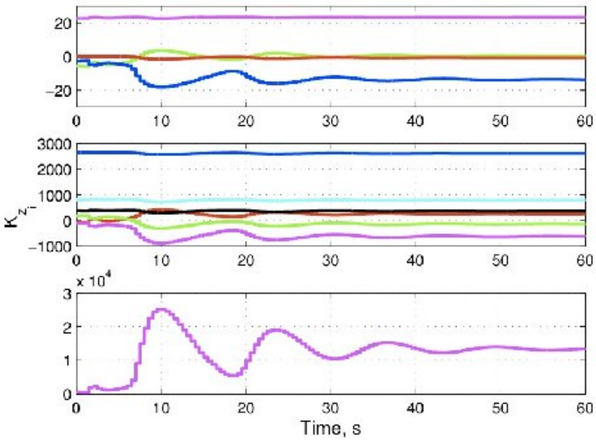

Fig. 8. Inner Loop Controller Gain K_z

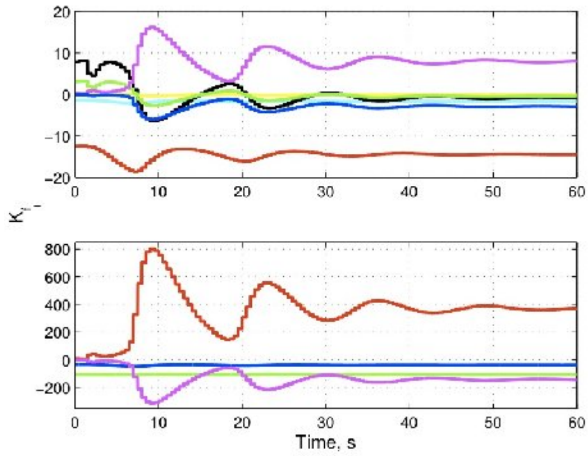

Fig. 9. Inner Loop Controller Gain K_f

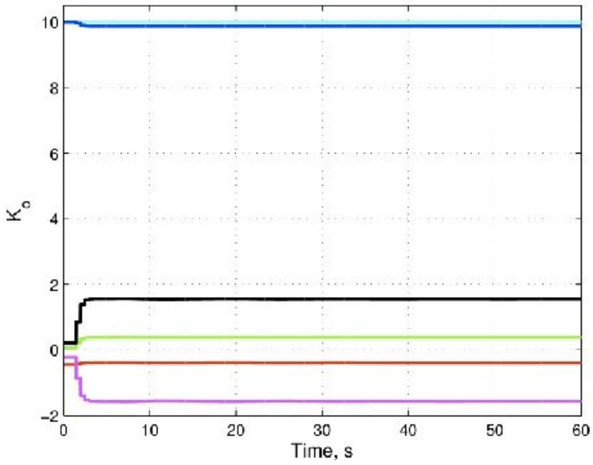

Fig. 10. Outer Loop Controller Gain K

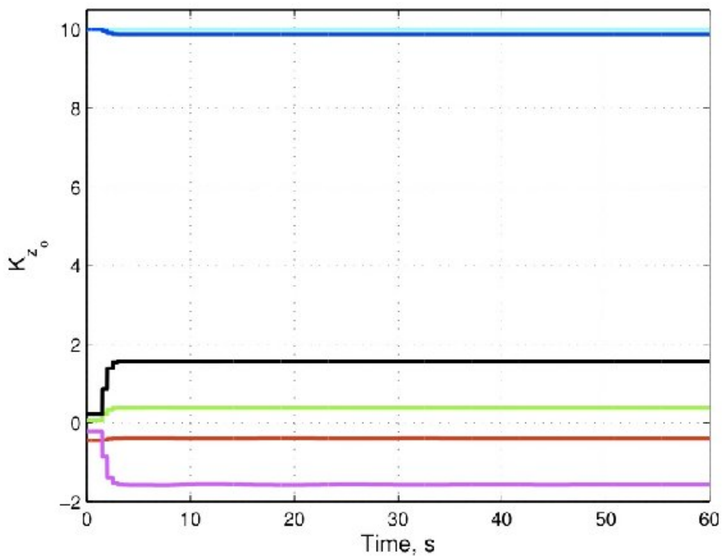

Fig. 11. Outer Loop Controller Gain K_z

Conclusions

In this paper, design of SDRE flight control system for a fixed-wing aircraft that operates in a nonlinear flight regime is presented. We introduce a dual-loop structure of the controller that allows decreasing dimensions of the state vectors and therefore reducing the order of SDC parameterization models. Flight control system utilizes a tracking algorithm and includes a nonlinear compensator for the gravity terms that are not taken into account by the parameterized models. The simulation results illustrate effectiveness of the proposed approach that utilized a single model the vehicle for the entire flight envelope, thus, eliminating need for linearization and gain scheduling.

Acknowledgments

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References

1. *Pearson J. D.* Approximation Methods in Optimal Control, Journal of Electronics and Control, 13, pp. 453-469, 1962.
2. *Mracek C. P.*, *Cloutier J. R.* and *D'Souza C. A.* A New Technique for Nonlinear Estimation, Proc. IEEE International Conference on Control Applications, pp. 338-343, Dearborn, MI, September 15-18, 1996.
3. *Cloutier J. R.*, *D'Souza C. N.* and *Mracek C. P.* Nonlinear Regulation and Nonlinear Hinf Control via the State-Dependent Riccati Equation Technique, Proc. International Conference on Nonlinear Problems in Aviation and Aerospace, Daytona Beach, FL, May 1996.
4. *Mracek C. P.* and *Cloutier J. R.* Control Designs for the Nonlinear Benchmark Problem via the State-Dependent Riccati Equation Method, International Journal of Robust and Nonlinear Control, 8, pp. 401-433, 1998.
5. *Cloutier J. R.*, *Mracek C. P.*, *Ridgely D. B.*, *Hammett K. D.* State Dependent Riccati Equation Techniques: Theory and Applications, Notes from the SDRE Workshop Conducted at the American Control Conference, Philadelphia, 1998.
6. *Mracek C. P.* and *Cloutier J. R.* Full Envelope Missile Longitudinal Autopilot Design Using the State-Dependent Riccati Equation Method, Proc. AIAA Guidance, Navigation, and Control Conference, New Orleans, LA, 1997.

7. *Cloutier J. R.* and *Stansbery D. T.* Nonlinear Hybrid Bank-to-Turn/ Skid-to-Turn Autopilot Design, Proc. AIAA Guidance, Navigation and Control Conference, Montreal, Canada, August 2001.
8. *Bogdanov A.* et al., SDRE Flight Control For X-Cell and R-Max Autonomous Helicopters, Proc. IEEE Conference on Decision and Control, IEEE, Atlantis, Paradise Island, Bahamas, pp. 1196-1203, 2003.
9. *Guo R.*, *Wu A.*, *Zhang X.* Improved SDRE Control for an Unmanned Helicopter Based on Multi-Timescale Dynamics Model, Proc. 8th World Congress on Intelligent Control and Automation, pp. 2476-248, Jinan, China, July 6-9, 2010.
10. *Voos H.* Nonlinear State-Dependent Riccati Equation Control of a Quadrotor UAV, Proc. IEEE International Conference on Control Applications, pp. 2547-2552, Munich, Germany, October 4-6, 2006.
11. *Parrish D. K., Ridgely D. B.* Attitude Control of a Satellite Using the SDRE Method, Proc. American Control Conference, pp. 942-946, Albuquerque, New Mexico, June, 1997.
12. *Luo W.*, *Chu Y-C.* Attitude Control Using the SDRE Technique, Proc. International Conference on Control, Automation, Robotics And Vision, pp. 1281-1286, Singapore, December 2002.
13. *Kayastha S.*, *Tekinalp O.*, *Ozgoren M. K.* Quaternion Based State Dependent Riccati Equation Control of a Satellite Camera on Piezoelectric Actuators, Proc. AIAA Astrodynamics Specialist Conference, Toronto, Canada, August 2010.
14. *Abdelrahman M.* et al. Magnetic Torque Attitude Control of a Satellite Using the State-Dependent Riccati Equation Technique", International Journal of Non-Linear Mechanics, vol. 46, pp. 75817771, 2011.
15. *Cimen T.* State-Dependent Riccati Equation (SDRE) Control: A Survey, Proc. 17th IFAC World Congress, pp. 3761-3775, Seoul, South Korea, 2008.
16. *Cimen T.* Systematic and Effective Design of Nonlinear Feedback Controllers via the State-Dependent Riccati Equation (SDRE) Method, Annual Reviews in Control, vol. 34, pp. 321751, 2010.
17. *Naidu D. S.* Optimal Control Systems, Boca Raton, Fla. : CRC Press, 2003.
18. *Anderson B.* and *Moore J. B.* Optimal control: Linear Quadratic Methods, Englewood Cliffs, N.J.: Prentice Hall. 8, 1989.
19. *Prach A.*, *Tekinalp O.* Development of a State Dependent Riccati Equation Based Tracking Flight Controller for an Unmanned Aircraft, Proc. AIAA Guidance, Navigation, and Control Conference, Boston, MA, August 19-22, 2013.

20. *Etkin B*. Dynamics of Atmospheric Flight, New York: Wiley, 1996.