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ESTIMATION OF THE LASER GYRO SYNCHRONIZATION ZONE DEPENDENCE ON THE CURVATURE RADIUS OF SPHERICAL MIRRORS

1. Introduction

Among the main types of the laser gyros that are widely used in practice, one can highlight the device based on a ring gas $He-Ne$ laser (the ratio of the isotope concentrations, $^{20}Ne:^{22}Ne=1:1$) with a flat N-mirror ($N = 3, 4$) resonator ensuring generation of a linearly polarized radiation in the sagittal plane. The laser, usually operating at wavelength $\lambda = 0.6328\times10^{-6}$ m, is pumped by a DC parallel discharge obtained by a common cathode and two anodes [1-3].

According to relations (5.55)–(5.57) from [3] and to expressions (6.45)-(6.47) from [4], when the currents are balanced in the discharge arms, the resonator is fine tuned to the center of the emission line and the losses are identical, the system of equations describing the dynamics of the dimensionless intensities I_j ($j=1, 2$) and the phase difference ψ of counter propagating waves of such a laser gyro can be written as

$$\begin{aligned} \dot{I}_1 &= (\alpha - \beta I_1 - \theta I_2) I_1 - 2r_2\sqrt{I_1 I_2}\cos(\psi + \varepsilon_2), \\ \dot{I}_2 &= (\alpha - \beta I_2 - \theta I_1) I_2 - 2r_1\sqrt{I_1 I_2}\cos(\psi - \varepsilon_1), \\ \dot{\psi} &= M\Omega + r_2\sqrt{I_2/I_1}\sin(\psi + \varepsilon_2) + r_1\sqrt{I_1/I_2}\sin(\psi - \varepsilon_1). \end{aligned} \tag{1}$$

In deriving these equations it was taken into account that the wave with $j=1$ propagates in the direction of the gyro rotation.

In system (1) α, β, θ are the Lamb coefficients that characterize the properties of the active medium; $M = (1 + K_a)M_g$ is the laser gyro scale multiplier, primarily determined by its geometrical component $M_g = 8\pi S/(\lambda L)$ and also taking into account the properties of the medium through a small parameter K_a (L is the perimeter of the axial contour; S is the covered area); Ω is the angular velocity of the device rotation in the inertial space; r_j and ε_j are the moduli and arguments of complex integral coefficients $r_j\exp\{i\varepsilon_j\}$ of the linear coupling of counterpropagating waves, characterizing their interaction through backscattering, absorption and transmission of radiation on the mirrors. (The relations for calculating the parameters α, β, θ of system (1) can be found, for example, in [5], and the parameter K_a – in [6]. An empirical formula for calculating K_a is presented in [3]. In addition, a set of expressions to estimate the parameters α, β, θ, K_a, r_j, ε_j is giving in [7]. These expressions are applicable for the case when the gyro operates at total pressures of the $He-Ne$ mixture from 1 to 5-6 Torr, and its resonator has the shape of an equilateral triangle or a square.)

In our paper [8], based on the analysis of (1) we obtained the formulas for calculating the parameters of the synchronization zone of the frequencies of counterpropagating electromagnetic waves generated in the laser gyro. These parameters are the coordinates $\Omega_{(-)}$ and $\Omega_{(+)}$ of the left and right boundaries of the synch Ω, the coordinate of its center $\Omega_{(0)} = (\Omega_{(+)} + \Omega_{(-)})/2$ and the half-width of this zone $\Omega_s = (\Omega_{(+)} - \Omega_{(-)})/2$. Relations obtained in [8] supplement the results of earlier theoretical studies [3, 9–16] and have the form

$$\Omega_{(\pm)} = \pm\frac{\sqrt{r_p^2 + \mu^2 r_m^2 \pm 2\mu(r_2^2 - r_1^2)}}{\sqrt{1-\mu^2}\,M},$$

$$\Omega_{(0)} = \frac{\sqrt{r_p^2 + \mu^2 r_m^2 + 2\mu(r_2^2 - r_1^2)} - \sqrt{r_p^2 + \mu^2 r_m^2 - 2\mu(r_2^2 - r_1^2)}}{2\sqrt{1-\mu^2}\,M}, \tag{2}$$

$$\Omega_s = \frac{\sqrt{r_p^2 + \mu^2 r_m^2 + 2\mu(r_2^2 - r_1^2)} + \sqrt{r_p^2 + \mu^2 r_m^2 - 2\mu(r_2^2 - r_1^2)}}{2\sqrt{1-\mu^2}\,M}.$$

In view of the condition $|r_2 - r_1| << (r_1 + r_2)/2$ (see, for example, [3]) implemented in practice, expressions (2) can be approximately rewritten in more compact form

$$\Omega_{(\pm)} = \Omega_{(0)} \pm \Omega_s, \qquad \Omega_{(0)} = \frac{\mu(r_2^2 - r_1^2)}{\sqrt{(1-\mu^2)(r_p^2 + \mu^2 r_m^2)}\,M},$$
$$\Omega_s = \frac{\sqrt{r_p^2 + \mu^2 r_m^2}}{\sqrt{1-\mu^2}\,M}, \tag{3}$$

where

$$r_p = \sqrt{r_1^2 + r_2^2 + 2 r_1 r_2 \cos\varepsilon_{12}}, \qquad r_m = \sqrt{r_1^2 + r_2^2 - 2 r_1 r_2 \cos\varepsilon_{12}},$$
$$\varepsilon_{12} = \varepsilon_1 + \varepsilon_2, \qquad \mu = \frac{2 r_1 r_2 \sin\varepsilon_{12}}{\alpha_m r_p} \qquad (|\mu| < 1), \tag{4}$$
$$\alpha_m = \alpha_p \frac{1-h}{1+h}, \qquad \alpha_p = \alpha = \frac{c}{L}(g - \Gamma), \qquad h = \frac{\theta}{\beta}.$$

Here r_p and r_m are the combinations of the parameters of the linear coupling of counterpropagating waves; α_p and α_m are, respectively, the inverse relaxation times of the sum and difference of the intensities of these waves; g is the unsaturated linear gain of the active medium; Γ is the resonator losses per trip; h is the parameter depending on the total pressure of the $He-Ne$ mixture [17]; μ is the quantity characterizing the effect of the active medium gain on the parameters of the synchronization zone. (Expressions (2) and (3) are valid under the condition of weak coupling of counterpropagating waves, which suggests that in the entire range of working discharge currents used in the laser gyros, the ratios r_p/α_p and r_m/α_m are much smaller than unity. In modern devices operating at sufficiently large excesses of the pump over the threshold [3], the above condition is usually satisfied.)

Based on the analysis of expressions (2)–(4) in [8], the following conclusions were drawn:

- in the general case of the asymmetric $(r_1 \neq r_2)$ linear coupling of the counterpropagating waves, the left and right boundaries of the synchronization zone of the laser gyro are located at different distances from the coordinate origin: $\Omega_{(+)} \neq -\Omega_{(-)}$. As a result, the center of this zone is shifted along the axis of the angular velocity Ω by the finite quantity $\Omega_{(0)} \neq 0$;
- with increasing the active medium gain g, the shift $\Omega_{(0)}$ of the center of the synchronization zone and its half-width Ω_s decrease, approaching asymptotically the established finite values

$$\Omega_{(0)}^{asymp} = 0, \qquad \Omega_s^{asymp} = \frac{r_p}{M} = \frac{\sqrt{r_1^2 + r_2^2 + 2 r_1 r_2 \cos \varepsilon_{12}}}{M}. \tag{5}$$

Restrictions on the amount of work [8] did not allow us to present the results of quantitative estimation of the dependence of the quantities $\Omega_{(+)}$, $\Omega_{(-)}$, $\Omega_{(0)}$, Ω_s on the active medium gain g for a particular laser gyro. Such estimation was performed in our following work [18] for the device with a four-corner square resonator which is formed by two flat mirrors and two identical spherical mirrors. The results, presented in [18], are in qualitative agreement with the known [19] – [22] experimental data obtained for the gyros with three-mirror resonators.

This paper is a supplement to work [18], and its purpose is: for the laser gyro (described in [18]) and for a simple special case of the symmetrical $(r_1 = r_2)$ linear coupling of the counterpropagating waves (when $\Omega_{(0)} = 0$), – to perform a quantitative estimation of the dependence of parameter Ω_s on the curvature radius of the spherical mirrors. The results of such estimation must be compared (qualitatively) with the known from [23] experimental data obtained for a gyro with a three-mirror resonator.

2. Laser gyro description and a set of expressions for calculating its parameters

Following [18], we will consider the laser gyro with a four-mirror square resonator having a nominal length of the arm $l = 50$ mm and a perimeter $L = 4\ell = 200$ mm. Such device is described in work [3], and it is characterized by the half-width of the synchronization zone, $\Omega_s \approx 0.05$ °/s. The angular resolution q_θ of the gyro is 2.61", and its geometrical scale multiplier $M_g = 496459$. The device operates at a total pressure of the $He - Ne$ mixture 6.5 Torr.

Using expressions (3) and (4), for the given gyro we will perform the quantitative estimation of parameter Ω_s under the condition that the curvature radius of the spherical mirrors of this device varies in the range from 1 to 4 m. In order not to present (with comments) the cumbersome formulas for calculation of the small parameter K_a, as well as expressions for estimates of β and θ, we will assume $M = M_g$, and in addition, set $h = \theta/\beta = 0.652$.

2.1. Relation for calculation of parameter Γ

In order to make use of expressions (3) and (4), we must first calculate the total resonator losses Γ for the given laser gyro. We assume that the resonator of this device is formed by two flat signal mirrors (M_1, M_2) and two spherical mirrors (M_3, M_4) with a radius of curvature R mounted on piezocorrectors (the mirrors are numbered clockwise). For the flat mirrors M_1 and M_2 we have specified the following energy parameters: integral coefficient K^f_{scat} of light scattering into the full solid angle 4π sr; absorption losses Γ^f_{absorp}; and useful transmission losses Γ^f_{transm}. For the spherical mirrors M_3 and M_4 we have specified the integral light scattering coefficient K^s_{scat}, and absorption losses Γ^s_{absorp}. Let $K^f_{scat} = 5\times10^{-6}$, $\Gamma^f_{absorp} = 55\times10^{-6}$, $\Gamma^f_{transm} = 60\times10^{-6}$, $K^s_{scat} = 10\times10^{-6}$, $\Gamma^s_{absorp} = 50\times10^{-6}$. Then, neglecting the small diffraction losses due to the presence of an aperture diaphragm in the gyro resonator, the desired formula for the calculation of Γ can be written in the form [18]

$$\Gamma = 2(K^f_{scat} + \Gamma^f_{absorp} + \Gamma^f_{transm} + K^s_{scat} + \Gamma^s_{absorp}) . \qquad (6)$$

With the given parameters of the mirrors, we find from (6) that $\Gamma = 360\times10^{-6}$.

2.2. Relations for calculation of parameters r_1, r_2 and ε_{12}

Now it is necessary to present the expressions for calculating the quantities r_1, r_2 and ε_{12}. Considering a simple case of the symmetrical ($r_1 = r_2$) linear coupling of counterpropagating waves, under condition that this coupling manifests itself in the maximum level (when on the length l of each resonator arm of the gyro there is an integer number of λ), on the base of formulas (21) and (22) from [18], we may write down the following relations:

$$r_1 = r_2 = 2\frac{c}{L}\Big\{a_f^2 + a_s^2 + 2\Big[a_f a_s \cos\left(\chi_f - \chi_s\right) + \left(a_f \sin\chi_f + a_s \sin\chi_s\right)\left(b_f + b_s\right)\Big] + \left(b_f + b_s\right)^2\Big\}^{1/2}, \quad (7)$$

$$\varepsilon_{12} = \pi - \mathrm{arctg}\frac{N_\varepsilon}{D_\varepsilon}, \quad (8)$$

where

$$N_\varepsilon = a_f^2 \sin 2\chi_f + a_s^2 \sin 2\chi_s + 2\Big[a_f a_s \sin\left(\chi_f + \chi_s\right) + \left(a_f \cos\chi_f + a_s \cos\chi_s\right)\left(b_f + b_s\right)\Big],$$

$$D_\varepsilon = a_f^2 \cos 2\chi_f + a_s^2 \cos 2\chi_s + 2\Big[a_f a_s \cos\left(\chi_f + \chi_s\right) - \left(a_f \sin\chi_f + a_s \sin\chi_s\right)\left(b_f + b_s\right)\Big] - \left(b_f + b_s\right)^2.$$

In the right-hand sides of these expressions, a_f and a_s are the moduli of local complex dimensionless coefficients of the counterpropagating waves coupling through backscattering of radiation, respectively, on the flat and spherical mirrors; χ_f and χ_s are the 'angles of scattering losses' on these mirrors; b_f are the moduli of local complex dimensionless coefficients of the counterpropagating waves coupling through absorption and transmission of radiation by the flat mirrors; b_s are the moduli of local complex dimensionless coefficients of the waves coupling through absorption by the spherical mirrors.

According to relations (8) from [18], the named quantities a_f, χ_f, b_f, a_s, χ_s, b_s may be calculated by the formulas

$$a_f = \frac{1}{2}\theta_f \sqrt{K_{scat}^f}, \quad \chi_f = \arcsin\sqrt{K_{scat}^f}, \quad b_f = \frac{1}{2}\theta_f \left(\Gamma_{absorp}^f + \Gamma_{transm}^f\right),$$

$$a_s = \frac{1}{2}\theta_s \sqrt{K_{scat}^s}, \quad \chi_s = \arcsin\sqrt{K_{scat}^s}, \quad b_s = \frac{1}{2}\theta_s \Gamma_{absorp}^s, \quad (9)$$

$$\theta_f = w_f/L, \quad w_f = \sqrt{w_f^{(x)} w_f^{(y)}}, \quad \theta_s = w_s/L, \quad w_s = \sqrt{w_s^{(x)} w_s^{(y)}},$$

$$w_f^{(z)} = \left(\frac{2\lambda l}{\pi}\right)^{1/2} \left[\frac{(4-7\zeta+2\zeta^2)^2}{4-(2-8\zeta+3\zeta^2)^2}\right]^{1/4},$$

$$w_s^{(z)} = \left(\frac{2\lambda l}{\pi}\right)^{1/2} \left[\frac{(4-3\zeta)^2}{4-(2-8\zeta+3\zeta^2)^2}\right]^{1/4}.$$

When using the two last expressions to estimate the quantities $w_f^{(z)}$ and $w_s^{(z)}$, it is needed to follow the rule: if the superscript is $z = x$ then $\zeta = \xi = pl$, where $p = 2(\sqrt{2}/R)$; but if the superscript is $z = y$ then $\zeta = \eta = ql$, where $q = \sqrt{2}/R$. Here p and q are the optical powers of the spherical mirrors, respectively, in the axial and sagittal planes; ξ and η are the small dimensionless parameters introduced for brevity.

In the right-hand sides of expressions (9), w_f and w_s are the effective half-widths of the Gaussian beam of the working laser gyro mode in its cross sections, where the flat and spherical mirrors are, respectively, placed; $w_f^{(x)}$, $w_s^{(x)}$ and $w_f^{(y)}$, $w_s^{(y)}$ are the half-widths of the Gaussian beam in the axial plane xz and sagittal plane yz in the above cross sections; θ_f and θ_s are half the angles at which one can see the light spots (of diameter $2w_f$ and $2w_s$) of the Gaussian beam on the surfaces of the flat and spherical mirrors, provided that they are observed from the centers of the same mirrors at a distance equal to L, in a situation when the axial contour of the gyro resonator is expanded in a straight line.

Relations (6)–(9) presented in this section for calculating the parameters Γ, r_1, r_2, ε_{12} of the considered laser gyro – allow us to perform a quantitative estimation of the dependence $\Omega_s = \Omega_s(R)$.

3. Quantitative estimation of the dependence of parameter Ω_s on curvature radius R of the spherical mirrors

For the laser gyro under consideration, the graphs of the dependence $\Omega_s = \Omega_s(R)$ are presented in fig. 1. These graphs are plotted by formulas (3), (4), (6)–(9) for three fixed values ($N_{rel} = 2, 4, 8$) of the relative excitation parameter $N_{rel} = g/\Gamma$ (where $\Gamma = 360\times10^{-6}$).

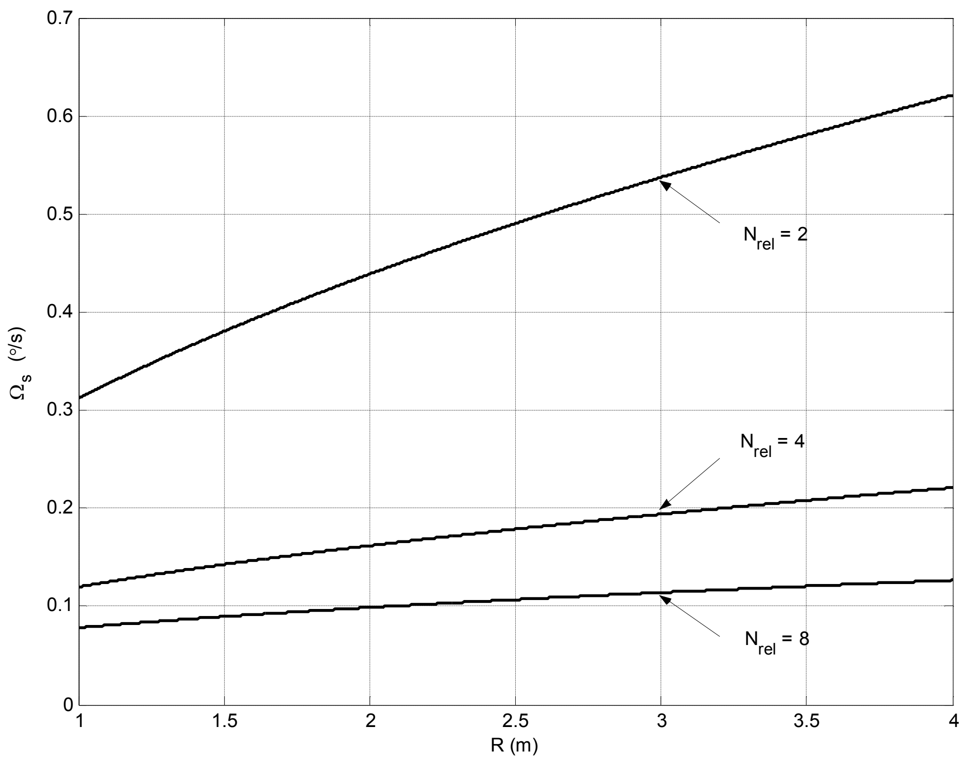

Fig. 1. The dependence of Ω_s (°/s) on R (m) for N_{rel} = 2, 4, 8

The upper graph in fig. 1 corresponds to the minimum value of the relative excitation parameter $N_{rel} = 2$, when the active medium gain g is $2 \times 360 \times 10^{-6}$. The middle one corresponds to intermediate value $N_{rel} = 4$, when g is $4 \times 360 \times 10^{-6}$. And, finally, the lower graph corresponds to the maximum value $N_{rel} = 8$, when $g = 8 \times 360 \times 10^{-6}$.

From analysis of these graphs it follows:

– with increasing the curvature radius R of the spherical mirrors, the half-width Ω_s of the laser gyro synchronization zone increases. Such character of the dependence $\Omega_s = \Omega_s(R)$ is in qualitative agreement with experimental data obtained in work [23] (see fig. 4 in [23]);
– with increasing the active medium gain g, the half-width Ω_s of the synchronization zone as well as its sensitivity to change in R decrease.

4. Conclusions

In this paper, we have considered a laser gyro with a four-mirror square resonator having a perimeter of 20 cm. For this device, using the basic expressions (3), (4) and auxiliary relations (6) – (9), we have performed a quantitative estimation of the dependence of the half-width Ω_s of the synchronization zone of the frequencies of counterpropagating waves on the curvature radius R of the spherical mirrors. The results of such estimation are in qualitative agreement with known experimental data [23] obtained for a gyro with a three-mirror resonator.

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