

UDC 629.783

DOI: <http://dx.doi.org/10.20535/2219-3804192018169623>**L. Ryzhkov**<sup>1</sup>, *professor*, **I. Karpenko**<sup>2</sup>, *student***COMPLEMENTARY FILTER SYNTHESIS of GYROSTABILIZER**

**Ua** В роботі розглядається синтез комплементарного фільтра, призначеного для визначення кутового положення платформи гіростабілізатора. Запропонована інженерна методика синтезу, яка дозволяє вибрати параметри фільтра та оцінити його ефективність. Розглянуто питання синтезу комплементарного фільтра у складі платформи. Показано, що динаміка платформи несуттєво впливає на якість роботи фільтра, що дозволяє виконувати синтез фільтра без врахування динаміки платформи.

**Ru** В работе рассматривается синтез комплементарного фильтра, предназначенного для определения углового положения платформы гиросtabilизатора. Предложена инженерная методика синтеза, позволяющая выбрать параметры фильтра и оценить его эффективность. Рассмотрен вопрос синтеза комплементарного фильтра в составе платформы. Показано, что динамика платформы несущественно влияет на качество работы фильтра, что позволяет выполнять синтез фильтра без учета динамики платформы.

**Introduction**

Complementary filters (CF) have become widely used to measure the angular position of moving objects in space [1-3] due to their simplicity and sufficiently high accuracy. Many models of such filters have been developed, depending on their purpose and the desired accuracy of measurements. In most cases, quaternion algebra is used. Due to the small dimensions, such systems can

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be effectively used as embedded systems of other devices. In particular, they can be used in gyrostabilizers to measure the angular position of the gyro stabilizer platform.

### Formulation of the problem

Consider the issue of the synthesis of the CF gyrostabilizer not as the register of the angle of the platform, but as an integral part of the contours of orientation and stabilization of this platform. We will take into account the smallness of the angles of rotation of the platform of a two-axis gyrostabilizer. This allows the CF to be substantially simplified by simplifying the procedure for integrating the output signal of the gyroscope, using the direct integration of this signal. It simplifies and compares the signals of the integrator and the accelerometer, for which it is not necessary to receive either quaternion, or a matrix of directional cosines.

### Synthesis of KF

At the output of the accelerometer, we have three signals, two of which (for small angles) are proportional to the angles of rotation of the platform relative to the horizon. Instead of comparing the output signals of the accelerometer and the integrator, it is advisable to form the correction signal in the form of a vector product [1].

$$\vec{e} = \frac{\vec{g}_{b^*}}{\|\vec{g}\|} \times \frac{\vec{g}_b}{\|\vec{g}\|},$$

where  $\vec{g}_{b^*}$  – real output signal of accelerometer;  $\vec{g}_b$  – calculated output signal of accelerometer, which uses the output signals of the CF.

For small angles, according to fig. 1, we have

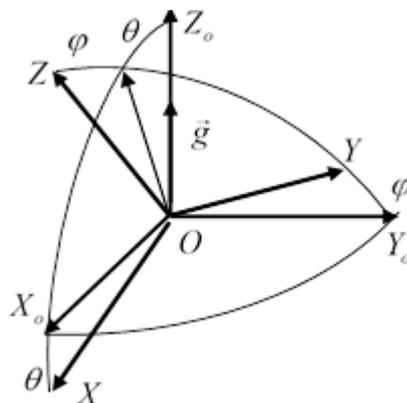


Fig. 1.

$$\mathbf{g}_{*n} = \begin{bmatrix} -\theta_* \\ \varphi_* \\ 1 \end{bmatrix}; \quad \mathbf{g}_n = \begin{bmatrix} -\theta \\ \varphi \\ 1 \end{bmatrix}.$$

Then the vector product is a vector

$$\begin{bmatrix} e_x \\ e_y \\ e_z \end{bmatrix} = \begin{bmatrix} g_{*ny}g_{nz} - g_{*nz}g_{ny} \\ g_{*nz}g_{nx} - g_{*nx}g_{nz} \\ g_{*nx}g_{ny} - g_{*ny}g_{nx} \end{bmatrix} = \begin{bmatrix} \varphi_* - \varphi \\ -\theta + \theta_* \\ 0 \end{bmatrix}.$$

That is, the components of this vector are proportional to the differences between the component vectors.

The generalized scheme of the CF is shown in fig. 2.

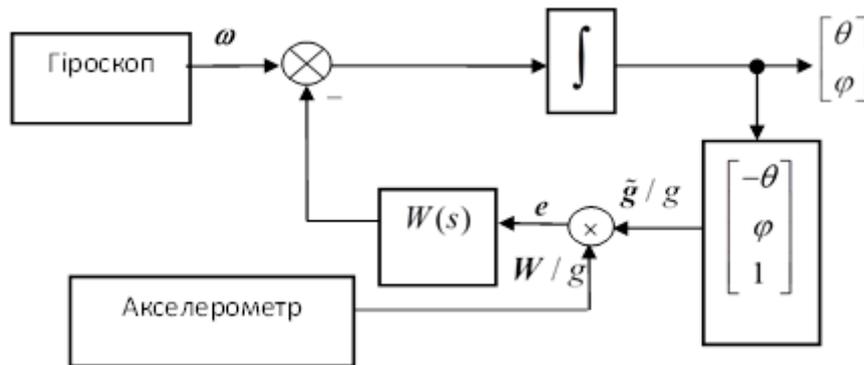


Fig. 2

In this formulation of the task, the control channels of the gyrostabilizer can be considered separately from each other. Consider the scheme shown in fig. 3.

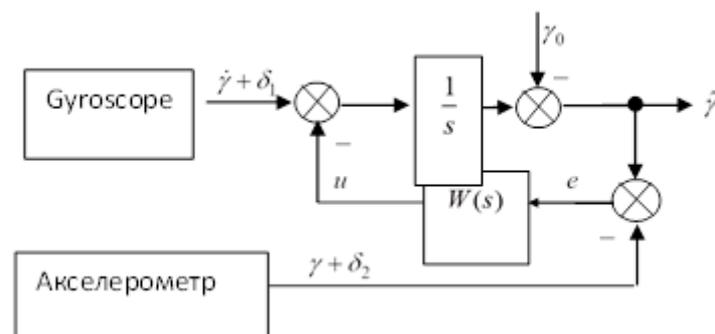


Fig.3

In the CF, signals from the angular velocity sensor (gyroscope) and accelerometer are used. The following notation has been entered:

$\gamma$  – angle that is measured;

$\hat{\gamma}$  – estimated value of angle  $\gamma$ ;

$\delta_1, \delta_2$  – errors of a gyroscope and an accelerometer;

$W(s)$  – transfer function of filter regulator;

$\gamma_0$  – the initial value of the angle  $\gamma$ , the introduction of which will be explained further.

Basic requirements for synthesis:

- to eliminate the infinite increasing of the error from integrating the zero signal of the integrator or to minimize its impact;
- to minimize the influence of the accelerometer errors due to linear accelerations of the base when changing the speed of the object;
- to minimize the effect of high-frequency noise of the accelerometer;
- to minimize the influence of initial conditions.

There is a relation

$$\hat{\gamma} = \gamma + \delta, \quad (1)$$

where the measurement error is equal to

$$\delta = \frac{1}{s + W(s)} \delta_1 + \frac{W(s)}{s + W(s)} \delta_2 - \frac{s}{s + W(s)} \gamma_0. \quad (2)$$

Consider the reason for entering the angle  $\gamma_0$ . This is explained by the fact that in the case of an initial angle, the accelerometer output signal and the integral of the gyroscope signal output will vary in magnitude of this angle even in the absence of interference due to the fact that the accelerometer measures the angle of deviation from the horizon and the integral of the output signal of the gyroscope is the angle of rotation object relative to the initial position. That is,

$$\gamma_{acc} = \gamma_{int\_gyro} + \gamma_0, \quad (3)$$

where  $\gamma_{acc}$  – output signal of accelerometer;

$\gamma_{int\_gyro}$  – signal of gyroscope after integration.

Since the initial conditions are more convenient to set on the integrator, in the scheme shown in fig. 3, instead of the angle  $\gamma_0$  of the accelerometer (object), the negative value of this angle is entered on the signal integrator from the gyroscope.

The transfer function  $W(s)$  will be taken in the form

$$W(s) = k_1 + \frac{k_2}{s}, \quad (4)$$

where  $k_1, k_2$  – coefficients.

Then

$$\delta = \frac{s}{s^2 + k_1s + k_2} \delta_1 + \frac{k_1s + k_2}{s^2 + k_1s + k_2} \delta_2 - \frac{s^2}{s^2 + k_1s + k_2} \gamma_0. \quad (5)$$

Consider the question of choosing the parameters  $k_1, k_2$  and assessing the quality of the CF.

From formula (4) we see that the constant component of the error from the drift of the gyroscope  $\delta_1$  will not be, but there is a question of numerical choice of parameters  $k_1$  and  $k_2$ .

Let's start with the consideration of the effects of the accelerometer errors, that is, from the analysis of the second component of the expression (5). To reduce this effect, it is desirable to reduce the parameters  $k_1, k_2$ . But this increases the time of the transient process. In addition, when increasing the parameter  $k_2$ , the filtering efficiency of the link  $\frac{k_1s + k_2}{s^2 + k_1s + k_2}$  decreases.

At the same time, in terms of reducing the influence of initial conditions, it is necessary to increase the parameter  $k_2$  to reduce the period of oscillations, that is, to reduce the time of the transient process.

Thus, to the choice of parameter  $k_2$ , demands are made both on its increase, and on its reduction.

We will accept the following parameters.

- drift angular velocity  $\omega_{dr} = 5 \cdot 10^{-4} \text{ 1/s}$ ;
- linear acceleration  $W = 0,2g = 2 \text{ м/с}^2$  during  $\tau = 5 \text{ sec}$ . (that is, the increase in velocity is 10 m/s). Note that such an acceleration corresponds (given in the corners) of perturbation  $W_* = 0,2 \text{ рад} = 11,46^\circ$ ;
- the amplitude of disturbances of the accelerometer is  $W = 0,1g$ , and the frequency of disturbances 10 Hz, that is 62,8 1/s;
- initial angle is  $\gamma_0 = 0,1$ , that is  $5,7^\circ$ .

Write the transfer function  $W(s) = \frac{k_1s + k_2}{s^2 + k_1s + k_2}$  in the form

$$W(s) = \frac{\xi ns + n^2}{s^2 + \xi ns + n^2},$$

where  $n = \sqrt{k_2}$  – frequency of own oscillations;

$\xi = \frac{k_1}{n}$  – relative coefficient of damping.

Assume  $k_2 = 0,001 \text{ s}^{-2}$ . Then  $n = \sqrt{k_2} = 0,0312 \text{ s}^{-1}$ . The period of its own oscillation is  $\frac{2\pi}{n} = 198,7 \text{ sec}$ . That is, the condition that the duration of the linear acceleration (5 sec.) was significantly less than the period of its own oscillations is fulfilled. Assume  $\xi = 1$ , that is  $k_1 = \xi n = 0,0312$ . Assume  $k_1 = 0,03 \text{ s}^{-1}$ . Note that with such parameters the link will be oscillating.

The reaction for a very short perturbation is equivalent to the response to the perturbation pulse of  $S = W_*\tau = 0,2 \cdot 5 = 1 \text{ s}^{-1}$ . We have

$$\delta_e = \frac{k_1 s + k_2}{s^2 + k_1 s + k_2} \delta_2 \approx \frac{s}{s^2 + k_1 s + k_2} k_2 \frac{\delta_2}{s} \approx \frac{s}{s^2 + k_1 s + k_2} \omega_e,$$

where  $\omega_e = k_2 \frac{\delta_2}{s} = k_2 \int_0^\tau \delta_2 dt \approx k_2 W_* \tau$  – equivalent angular velocity.

Thus, the effect of short-term acceleration is equivalent to the effect of an additional angular velocity  $\omega_e = 1 \cdot 10^{-3} \text{ s}^{-1}$ .

The maximum value of a variable  $\delta_e = \frac{s}{s^2 + k_1 s + k_2} \omega_e$  can be estimated by the formula

$$\delta_{e \max} = \frac{s}{s^2 + \xi n s + n^2} \omega_e \approx n \frac{\omega_e}{n^2} = \frac{\omega_e}{n} = \frac{1 \cdot 10^{-3}}{0.03} = 1,9^\circ.$$

Note, that a similar maximum error in the absence of CF is equal to, approximately  $W_* = 11,46^\circ$ .

A similar one can find the maximum deviation from the angular velocity of drift

$$\delta_{\omega_{dr}} = \frac{s}{s^2 + \xi n s + n^2} \omega_{dr} \approx \frac{\omega_{dr}}{n} = \frac{5 \cdot 10^{-4}}{0.03} = 0,85^\circ.$$

Simulation results (depending on time) are presented in fig. 4. Here is a continuous curve – the reaction to the angular velocity of drift in the absence of a filter, the point curve – the reaction to the angular velocity of drift in the presence of a filter, the dash curve – the reaction to linear acceleration (the time is given in seconds, the angles – in degrees).

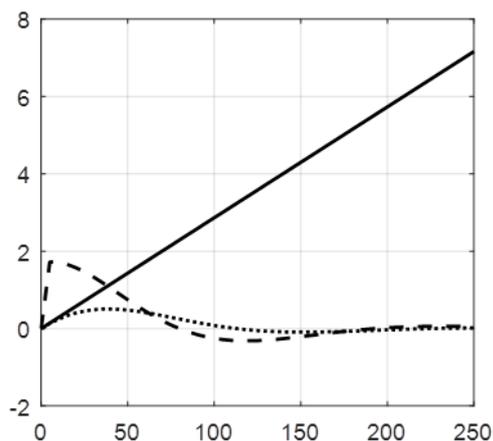


Fig. 4.

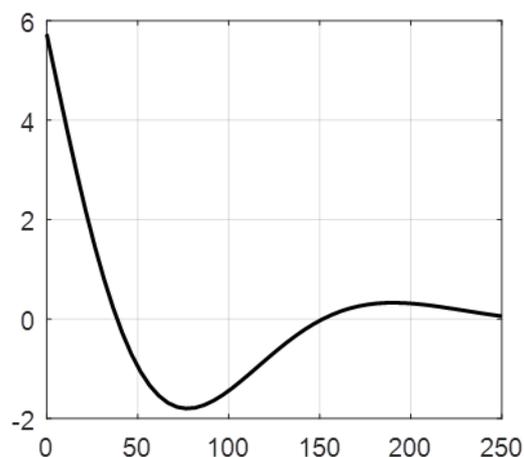


Fig. 5.

We see that the error from the drift of the gyroscope goes to zero, that is, the main purpose of the CF is fulfilled.

Given that the frequency of the own oscillations of the filter is very small, the filter almost completely eliminates the effect on the accuracy of the filter

operating on the accelerometer harmonic perturbations of the type of vibration, whose frequencies are tens of times greater than the frequency of the own oscillations of the filter.

Results of simulation of the third component of the error of the expression (4) are shown in fig. 5.

We see that the error from the initial conditions goes to zero, but because of the small value of the coefficient, this occurs over several tens of seconds.

Consider now the functioning of the CF as part of the control circuit of the gyrostabilizer. We record the equation of motion of the gyro stabilizer platform. Take control of the platform in the form

$$M_k(s) = r_1 \hat{\gamma} + r_2 (\omega + \delta_1), \quad (6)$$

where  $r_1, r_2$  – coefficients.

That is, the angle is calculated by a complementary filter, and the angular velocity is measured by a gyroscope. We will write the equation of the platform movement in the form

$$Is^2\gamma = -M_k(s) + M, \quad (7)$$

Where  $I$  – moment of inertia of the platform;

$M$  – moment of disturbances.

The influence of the error  $\delta_1$  in the platform equation (6) can be estimated as follows. It will lead to additional error  $\gamma_{\delta_1} \approx -\frac{r_2}{r_1} \delta_1$ . Let's accept  $\frac{r_2}{r_1} = 0,35 s$ .

Then, with gyroscope drift  $\omega_{dr} = 5 \cdot 10^{-4} 1/s$ , the error will equal the  $\gamma_{\delta_1} \approx -0,25 \cdot 5 \cdot 10^{-4} = -1,25 \cdot 10^{-4}$  rad. That is, the influence of the error  $\delta_1$  in the composite moment of damping in the equation of motion of the platform can be neglected.

In modeling we consider only the question of the influence of the initial angle  $\gamma_0$ . The simulation scheme of the platform in Simulink environment is shown in fig. 6. It corresponds to the equation  $s^2\gamma = -5s\gamma - 20\hat{\gamma}$ .

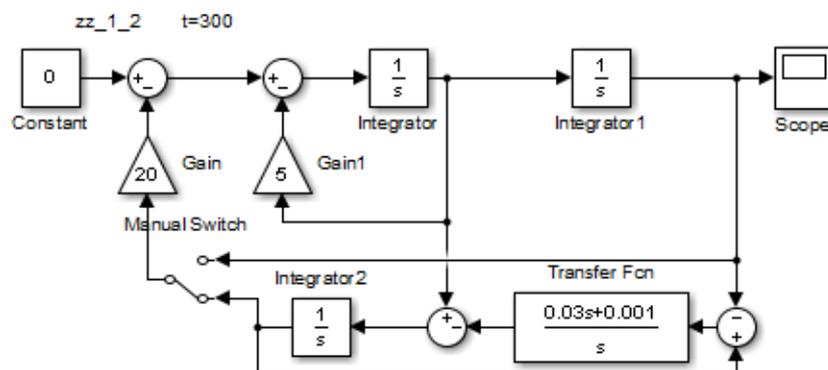


Fig. 6.

Fig. 7 shows the transient process in the absence of CF. In Fig. 8 – when using it. We see that the time of the transition process has increased significantly.

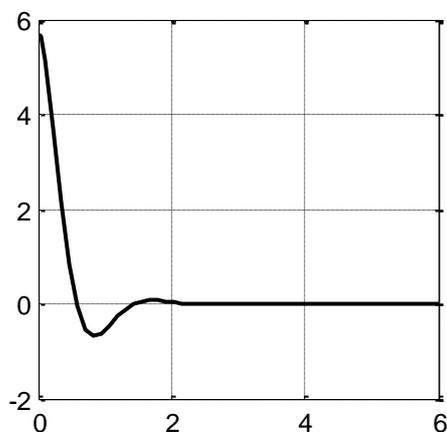


Fig. 7.

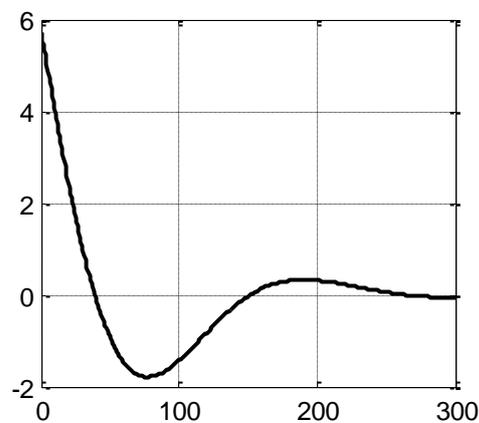


Fig. 8/

Fig. 7 and fig. 5 practically coincide. This means that the transient process in the platform is determined not by the parameters of the platform itself, but by the parameters of the CF. From this, one can also draw a conclusion that the synthesis of CF parameters can be performed without taking into account the dynamics of the platform.

### Conclusions

The use of CF is an effective means of improving the dynamic characteristics of a mobile gyrostabilizer. When used in systems of this type, simplified circuit-technical decisions of the CF can be used. Synthesis of CF parameters can be performed without taking into account the platform dynamics.

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