УДК 629.58:681.5

DOI: http://dx.doi.org/10.20535/2219-3804192018169438

O. Ivanova[1], *senior lecturer,* **D. Ilchishyna**[2], *docent*, **S. Chernenko**[3], *student*

THE EXTENDED THEORY OF ELASTIC BODIES COLLISION

Ua У роботі проводиться дослідженні залежності сили динамічної взаємодії твердих тіл під час удару від місцевого стиснення, що отримана відповідно до розширеної теорії удару.

Результати теоретичного дослідження порівнюються із результатами експерименту, що проведений раніше іншими авторами.

Можна зробити висновок, що результати, які отримані за розширеною теорією, якісно краще узгоджуються із даними експерименту, ніж результати отримані за класичною теорією Г. Герца.

[1] *Igor Sikorsky Kyiv Polytechnic Institute, Department of flight control systems*
[2] *Igor Sikorsky Kyiv Polytechnic Institute, Department of flight control systems*
[3] *Igor Sikorsky Kyiv Polytechnic Institute, Institute of Mechanical Engineering*

Ru В работе проводится исследование зависимости силы динамического взаимодействия твердых тел при ударе от местного сжатия, полученной согласно расширенной теории удара.

Результаты теоретического исследования сравниваются с результатами эксперимента, проведенного ранее другими авторами.

Можно заключить, что результаты, полученные по расширенной теории, качественно лучше согласуются с данными эксперимента, чем результаты, полученные по классической теории Г. Герца.

Introduction

A method for clarifying the extended theory of impact, known from [1], is considered in the monograph [2]. This method takes into account the dynamic terms in the Lamé equation, to which the solution of the dynamic problems of the elasticity theory is narrowed down. Thus, one of the main simplifying assumptions of the classical G. Hertz theory is eliminated, according to which the inertial forces have little effect on the local distribution of stresses in the contact area. Here, the strength of the dynamic interaction P is determined by the difference

$$P = 2P_{н} - P_{к} \qquad (1)$$

where $P_{н}$ – is the strength of the dynamic interaction corresponding to the classical theory of G. Hertz, $P_{к}$ – the extended theory.

However, equality (1) generally corresponds to the mechanical model described by the shock theory developed by G. Hertz. The difference from the G. Hertz model here, as can be seen from [2], is the introduction of local inertia forces arising from the deformation of an elastic element included in a mechanical model (the model corresponds to the theory of G. Hertz).

Statement of the problem

The purpose of this paper is to study the properties of the curve $P = P(\alpha)$ (dependence of the dynamic interaction force P and colliding bodies on local compression α), obtained according to the extended impact theory, on the basis of the approximate dependencies indicated in papers [2, 3, 4].

The results of experiments leading to the determination of the exact dependence of $P(\mathrm{t})$ and $\alpha(\mathrm{t})$ are presented in papers [1, 5]. This allows us to compare theoretical and experimental results.

Research

In the classical theory of G. Hertz, the relationship between the interaction force and the relative displacement of the centers of inertia of bodies, obtained by static compression, is preserved even in the case of a collision, i.e.

$$\alpha_н = kP_н^{\frac{2}{3}} \tag{2}$$

here $\alpha_н = \alpha_н(t)$; $P_н = P_н(t)$.

Substitution $t = \tau \dfrac{m^{\frac{2}{5}} k^{\frac{3}{5}}}{V_o^{\frac{1}{5}}}$ (τ – non-dimensional time) allows presenting values also α, P in dimensionless form

$$\alpha_н = \left(\frac{mV_o^2}{k}\right)^{\frac{2}{5}} k\varphi_н(\tau); \qquad P_н = \left(\frac{mV_o^2}{k}\right)^{\frac{3}{5}} f_н(\tau).$$

Based on (2) we have

$$f_н(\tau) = [\varphi_н(\tau)]^{\frac{3}{2}}, \tag{3}$$

where $f(\tau)$ – is the dimensionless dynamic interaction force;

$\varphi(\tau)$ – is the relative displacement of the centers of inertia of bodies.

The functional connection (3) is represented by curve 1 on fig. 1.

The classical quasi-static theory of G. Hertz considers an absolutely elastic impact; therefore, two branches of curve 1 found for two stages of the impact coincide.

Dependence of P_k from $\alpha_к$ obtained under the extended theory in dimensionless quantities is shown in Fig. 1, curve 2 has the form of a loop, the branches of which correspond to an increase and then a decrease in strength and deformation during the collision process.

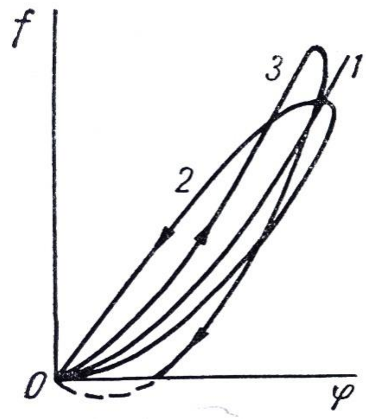

Fig. 1.

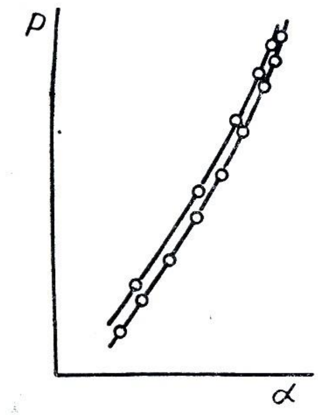

Fig. 2.

This curve can be called the “reversed hysteresis loop”, since its branch corresponding to the load is located below the unloading branch. It follows that the function $f_к = f_к(\varphi)$ needs corrections indicated in paper [3].

Using the results [2], we can put down the equation (1) as follows

$$f = f(\varphi) = 2f_н - f_к. \quad (4)$$

Curve 3 describes “hysteresis loop”. The term “hysteresis loop” is used conditionally, since this hysteresis is not associated with the internal properties of the substance, but with local inertia forces. The dimensionless projection duration found from the graph in fig. 1 is equal to $\tau_{\max} \cong 3,203$. The part of curve 3, indicated by the dashed line, corresponds to the negative value of the interaction force P. In the case of adhesion forces absence between the colliding bodies, the impact ends at $P = 0$.

The results of the experiment related to the determination of the exact dependence of the dynamic interaction force on time are presented in papers [1, 5]. The impact force was measured using piezoelectric sensors. The data obtained on the basis of the experiment, as well as the values of α found from the obtained values of P from the equation of motion, allowed us to construct the so-called indicator diagram, which determines the dependence of P on α (fig. 2).

Comparing curve 3 in fig. 1 and the indicator diagram in fig. 2, we can conclude that curve 3, obtained by the extended theory [2], qualitatively conforms with the dependence of P on α, constructed from the experimental data.

Conclusions

The established dependence of P on α in the presence of the influence of inertia forces is represented by a curve similar to the hysteresis loop. However, as stated above, its origin is different.

The obtained results are mainly of qualitative importance, since they are based on the approximate dependencies mentioned above. Making clarifications to these dependencies may change the quantitative description of a phenomenon.

References

1. *Goldsmith V.* Impact, M., Stroyizdat, 1965.
2. *Kilchevsky N. A.*, Dynamic Contact Compression of Solids. Impact., K., Publishing House "Naukova Dumka", 1976.
3. *Ilchishina D. I.*, Shalda L. M. Surface waves of collision of elastic bodies and generalization of the theory of impact G. Hertz, VIII International Conference “Hydrotechnology, navigation, traffic control and design of aerospace equipment”, K., 2011.
4. *Ivanova O. M.*, Ilchishina D. I., Effect of Rayleigh waves on the impact process, MGS, No. 28, 2014.

5. *Crook A. W.* A study of some impacts between bodies by a piezo-electric method, London, Proc. Roy. Soc., ser. A, vol. 212, 1952.