UDC 536.3

DOI: http://dx.doi.org/10.20535/2219-38041182018132575

S. I. Trubachev[1], *PhD, Associate professor,*
O. M. Alekseychuk[2]**,** *PhD, Associate professor*

STRESS-DEFORMED STATE OF COMPOSITE SHELLS WITH FILLER

Ua У роботі представлено методику розрахунку композитних шаруватих оболонок з жорстким заповнювачем, яка заснована на вариаційно-сітковому підході формування необхідних функціоналів із їх подальшою мінімізацією методом покоординатного спуску. Побудовані алгоритми чисельного розрахунку відрізняються стійкістю і економічністю із точки зору обчислювальних ресурсів. На прикладі чисельного експерименту було досліджено збіжність рішень.

Ru В работе представлена методика расчета композитных слоистых оболочек с жестким заполнителем, основанная на вариационно-сеточном подходе формирования необходимых функционалов с их последующей минимизацией методом покоординатного спуска. Построенные алгоритмы численного расчета отличаются устойчивостью и экономичностью с точки зрения вычислительных ресурсов. На примере численного эксперимента была исследована сходимость решений.

Formulation of the problem

Laminated shells, that are made of high-strength composite materials with different layers laying, have found wide application in aircraft engineering as

[1] *NTUU «Igor Sikorsky Kyiv Polytechnic Institute», Department of*
[2] *NTUU «Igor Sikorsky Kyiv Polytechnic Institute», Department of Theoretical Mechanics*

elements of bearing surfaces of aircrafts, as well as in many other industries. Thus, the improvement of calculating methods for heterogeneous layered structures is very actual task. The numerical methods of investigation are allow to predict the possible destruction of the structure and have the same actuality as the methods of nondestructive testing, flaw detection of structures with the help of hardware methods.

The designations are taken from the classical theory of shells [1].

The point's displacements of the shell supporting layers can be represented in the following form

$$\begin{gathered} u^{(n)}(\alpha_1,\alpha_2,z)=u_0^{(n)}(\alpha_1,\alpha_2)+z^{(n)}\theta_1^{(n)}(\alpha_1,\alpha_2)\,, \\ v^{(n)}(\alpha_1,\alpha_2,z)=v_0^{(n)}(\alpha_1,\alpha_2)+z^{(n)}\theta_2^{(n)}(\alpha_1,\alpha_2)\,, \\ w^{(n)}(\alpha_1,\alpha_2,z)=w_0^{(n)}(\alpha_1,\alpha_2), \quad n=1,2\,, \end{gathered} \tag{1}$$

where $\theta_1^{(n)}$ and $\theta_2^{(n)}, n=1,2$ – are the normal's angles of rotation in the carrier layers, respectively in the planes $\alpha_1=\text{const}, \alpha_2=\text{const}$:

$$\begin{gathered} \theta_1^{(n)}(\alpha_1,\alpha_2)=-\frac{1}{A_1}\frac{\partial w_0^{(n)}}{\partial\alpha_1}+\frac{u_0^{(n)}}{R_1}; \\ \theta_2^{(n)}(\alpha_1,\alpha_2)=-\frac{1}{A_2}\frac{\partial w_0^{(n)}}{\partial\alpha_2}+\frac{v_0^{(n)}}{R_2} \end{gathered} \tag{2}$$

$u_0^{(n)}, v_0^{(n)}, w_0^{(n)}, n=1,2$ – are the points displacement component of the carrier layers middle surface in direction of the coordinate axes in adopted coordinate system.

Solution method

We take the nonlinear distribution of displacements for the filler.

$$\begin{gathered} u^{(n)}(\alpha_1,\alpha_2,z)=u_0^{(n)}(\alpha_1,\alpha_2)+\sum_{m=1}^{3}u_i(\alpha_1,\alpha_2)z^m\,; \\ v^{(n)}(\alpha_1,\alpha_2,z)=v_0^{(n)}(\alpha_1,\alpha_2)+\sum_{m=1}^{3}v_i(\alpha_1,\alpha_2)z^m\,; \\ w^{(n)}(\alpha_1,\alpha_2,z)=w_0^{(n)}(\alpha_1,\alpha_2)+\sum_{m=1}^{3}w_i(\alpha_1,\alpha_2)z^m\,. \end{gathered} \tag{3}$$

We can represent the stresses in the carrier layers in the form [1]

$$\{\sigma^{(n)}\}=[G^{(n)}]\{\varepsilon^{(n)}\}, \quad n=1,2, \tag{4}$$

where $\{\sigma^{(n)}\}=\{\sigma_1^{(n)},\sigma_2^{(n)},\tau_{12}^{(n)}\}$ – are the components of the stress tensor;

$\{\varepsilon^{(n)}\}=\{\varepsilon_1^{(n)},\varepsilon_2^{(n)},\gamma_{12}^{(n)}\}$ – are the components of the deformations tensor.

$$[G^{(n)}]=\begin{pmatrix} g_{11}^{(n)} & \mathrm{g}_{12}^{(\mathrm{n})} & g_{13}^{(n)} \\ \ldots & \mathrm{g}_{22}^{(\mathrm{n})} & g_{23}^{(n)} \\ \ldots & \ldots & g_{33}^{(n)} \end{pmatrix}$$

is the matrix of elastic coefficients. For an isotropic material of carrier layers we have

$$g_{11}^{(n)}=g_{22}^{(n)}=\frac{E_n}{1-v_2^{(n)}};\quad g_{12}^{(n)}=\frac{E_n v_n}{1-v_n^2};\quad g_{13}^{(n)}=g_{23}^{(n)}=0;\quad g_{33}^{(n)}=G^{(n)}. \tag{5}$$

For a filler as a vector component $\{\sigma^{(3)}\}$ and $\{\varepsilon^{(3)}\}$ we have

$$\{\sigma^{(3)}\}=\{\tau_{13}^{(3)},\tau_{23}^{(3)},\sigma_3^{(3)}\},\qquad \{\varepsilon^{(3)}\}=\{\gamma_{13}^{(3)},\gamma_{23}^{(3)},\varepsilon_3^{(3)}\}. \tag{6}$$

According to Hooke's law we have:

$$\{\sigma^{(3)}\}=[G^{(3)}]\{\varepsilon^{(3)}\}. \tag{7}$$

In the case of coincidence of the orthotropic axes with coordinate lines, the matrix $[G^{(3)}]$ has a diagonal structure.

$$[G^{(3)}]=[g_{55}^{(3)},g_{44}^{(3)},g_{33}^{(3)}], \tag{8}$$

where for a material of uniform thickness we have

$$g_{55}^{(3)}=G_{13}^{(3)};\qquad g_{44}^{(3)}=G_{23}^{(3)};\qquad g_{33}^{(3)}=E_z^{(3)}. \tag{9}$$

The potential energy of deformation of a three-layer plate is equal to the sum of the potential deformation energies of the carrier layers and filler:

$$u=\frac{1}{2}a(\upsilon,\upsilon)=\frac{1}{2}\sum_{i=1}^{3}\int_{V(i)}\{\epsilon^{(i)}\}^T\{\sigma^{(i)}\}dV_i\,. \tag{10}$$

Here is

$$\{\sigma^{(n)}\}=\{\sigma_x^{(n)},\sigma_y^{(n)},\tau_{xy}^{(n)}\};\quad \{\epsilon^{(n)}\}=\{\epsilon_x^{(n)},\epsilon_y^{(n)},\gamma_{xy}^{(n)}\},\quad n=1,2$$

$$\{\sigma^{(3)}\}=\{\tau_{xx}^{(3)},\tau_{yx}^{(3)},\sigma_z^{(3)}\};\quad \{\epsilon^{(3)}\}=\{\gamma_{xz}^{(3)},\gamma_{yz}^{(3)},\epsilon_z^{(3)}\}.$$

To approximate the deflection of the n-th thin carrier layer within each subdomain, we use the function [2]

$$\begin{aligned} w_{0h}^{(h)} &= w_1^{(n)}L_1+w_2^{(n)}L_2+w_3^{(n)}L_3+a_1^{(n)}L_1^2L_2+a_2^{(n)}L_1^2L_3+ \\ &+a_3^{(n)}L_2^2L_1+a_4^{(n)}L_2^2L_3+a_5^{(n)}L_3^2L_1+a_6^{(n)}L_3^2L_2+2a_7^{(n)}L_1L_2L_3\,, \end{aligned} \tag{11}$$

where

$$a_1^{(n)}=w_1^{(n)}-w_2^{(n)}-b_3\varphi_1^{(n)}-c_3\psi_1^{(n)};$$

$$a_2^{(n)}=w_1^{(n)}-w_3^{(n)}+b_2\varphi_1^{(n)}-c_2\psi_1^{(n)};$$

$$a_3^{(n)}=w_2^{(n)}-w_1^{(n)}+b_3\varphi_2^{(n)}+c_3\psi_2^{(n)};$$

$$a_4^{(n)}=w_{21}^{(n)}-w_3^{(n)}-b_1\varphi_2^{(n)}-c_1\psi_2^{(n)};$$

$$a_5^{(n)} = w_3^{(n)} - w_1^{(n)} - b_2\varphi_3^{(n)} - c_2\psi_3^{(n)} \,;$$

$$a_6^{(n)} = w_3^{(n)} - w_2^{(n)} + b_1\varphi_3^{(n)} + c_1\psi_3^{(n)} \,;$$

$$a_7^{(n)} = \frac{1}{4}\sum_{s=1}^{6} a_s^{(n)} \,.$$

Herein L_i, i = 1, 2, 3 – L are the coordinates that were defined by relations [3, 4]:

$$L_i = \frac{1}{2\Delta}(a_i + b_i x + c_i y)\,, \tag{13}$$

where

$$a_1 = x_2 y_3 - y_2 x_3\,; \quad b_1 = y_2 - y_3\,; \quad c_1 = x_3 - x_2\,. \tag{14}$$

Moving medial surfaces of carrier layers and fillers are represented as linear polynomials

$$u_{0h}^{(n)} = u_1^{(n)} L_1 + u_2^{(n)} L_2 + u_2^{(n)} L_2\,; \tag{15}$$

$$v_{0h}^{(n)} = v_1^{(n)} L_1 + v_2^{(n)} L_2 + v_2^{(n)} L_2\,;.\, w_h^{(3)} = w_1 L_1 + w_2 L_2 + w_3 L_3\,. \tag{16}$$

Then the total potential energy of each triangle can be written in the local coordinate system in the form of:

$$\hat{э}\,(\hat{v}_h) = \sum_{n=1}^{3}\hat{U}_h^{(n)} - \sum_{n=1}^{2}\sum_{i=1}^{3}\left(F_{xi}^{(n)}\hat{u}_i^{(n)} + F_{yi}^{(n)}\hat{v}_i^{(n)} + F_{zi}^{(n)}\hat{w}_i^{(n)} + F_{\varphi i}^{(n)}\hat{\varphi}_i^{(n)} + F_{\psi i}^{(n)}\hat{\psi}_i^{(n)}\right) \tag{17}$$

$$э\,(v_h) \equiv э\,(\vec{v}) = \frac{1}{2}(K\vec{v}, v) - (\vec{f}, \vec{v}); \vec{v}, \vec{f} \in R\,, \tag{18}$$

where is a positive definite symmetric N- ordered matrix K is a vector of a node external load with N dimension. Then the problem of determining the stress-strain state of structures can be represented in the form of minimizing the quadratic functional

$$\vec{u} \in R^n;\; э\,(\vec{u}) = \inf_{\vec{v}\in R^N} э\,(\vec{v})\,. \tag{19}$$

The minimization of the functional is carried out by the method of coordinate wise descent [5], where the approximation vector is sought in the form

$$\vec{\upsilon}^{k+1} = \vec{\upsilon}^{k} + \lambda_i^{k+i}\vec{e}_i, \quad i = 1, \dots, N\,, \tag{20}$$

where $\vec{e}_i$ is a single vector in directional $\upsilon_i^k, \lambda_i^{k+1}$ – is a step.

It should be noted that the method of coordinate-wise descent is differs greatly in the numerical calculation.

A pliable-displacement plate was considered as a test case, Mindlin-Reisner’s non-classical theory of plates was applied for its calculation [6].

$$Gh_s\left(\Delta w+\Phi\right)+q=0. \tag{21}$$

After scheduling the load $q(x_1, x_2)$ into a double row we have:

$$q=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}q_{mn}\sin\left(\alpha_m x_1\right)\sin\left(\beta_n x_2\right). \tag{22}$$

Where

$$q=\frac{4}{\ell_1\ell_2}\int_0^{\ell_1}\int_0^{\ell_2}q\left(x_1, x_2\right)\sin\left(\alpha_m x_1\right)\sin\left(\beta_n x_2\right)=q_{mn}=\frac{16q_0}{\pi^2 mn}, \tag{23}$$
$$m, n=1, 3, 5, \ldots.$$

The equality of deflections will be obtained after integration:

$$w=\frac{1}{K}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{q_{mn}}{\left(\alpha_m^2+\beta_n^2\right)}\left[1+\frac{K}{Gh_s}\left(\alpha_m^2+\beta_n^2\right)\right]\sin\left(\alpha_m x_1\right)\sin\left(\beta_n x_2\right). \tag{24}$$

Taking into account that $K=\dfrac{Eh^3}{12(1-\nu^2)}$ and $G=\dfrac{E}{2(1+\nu)}$, we have $\dfrac{K}{Gh_s}=\dfrac{h^2}{5(1-\nu)}$.

The Reissner's theory equations were applied when we calculating the composite with Navier boundary conditions. It is assumed that the plate is homogeneous and completely consists of a polymer: $E=E_в=E_н=E_c=7{,}9$ MPa, $\nu=\nu_c=\nu_в=\nu_н=0{,}41$, $\rho=\rho_c=\rho_в=\rho_н=960\cdot10^{-9}$ kg/m^3; the thickness of the plate $h=h_c=h_в=h_н=7{,}4$mm, plane dimensions $l_1=1620$ mm, $l_2=810$ mm; distributed load $q_0=p=0{,}25\cdot10^{-9}$ N/mm^2 [6].

In order to confirm the correctness of the results, a numerical calculation was made by using the approximations (3) and (11) together with the analytical solution. Tab. 1 shows the convergence of results, and in Tab. 2 the corresponding results of the analytical and numerical solution are compared, and the calculation error was determined.

***Table* 1.**

Parameters of finite element breakdown for numerical calculation of limiting cases

	Number FE	w [mm]
grid 1	123	3,261
grid 2	234	3,302
grid 3	728	3,411
grid 4	3556	3,464
grid 5	7832	3,495
grid 6	21422	3,501

***Table* 2.**

Results of calculations in the limiting cases under the static load of the plate

	Numerical solution w_{num} [mm]	Reisner-Mindlin w_{an} [mm]
	3,501	3,5722
δ [%]	2,04	

Tab. 2 shows that the error between the analytical and numerical solution is less than 5%. This suggests the effectiveness of the proposed approach.

Conclusions

An effective method for calculating of layered shell structures with filler was proposed in this paper. Nonlinear functions for displacements were used for the filler in contrast to the direct normal hypothesis. The method is based on the variational-grid approach for the formation of functionals, with its subsequent minimization by method of coordinate-descent. This approach allows to avoid the known difficulties associated with the formation, storage and operation with global stiffness matrices.

References

1. *Алфутов Н. А*., Зиновьев П. А., Попов Б. Г. Расчет многослойных пластин и оболочек из композиционных материалов. М.: Машиностроение, 1984.-264с.
2. *Василенко Н. В*., Бабенко А. Е., Трубачев С. И. Расчет трехслойных пластин с мягким заполнителем, подверженных статическим и динамическим нагрузкам // Проблемы прочности.- 1989.-№2. - С.71-74.
3. *Зенкевич О*. Метод конечных элементов в технике.-М.: Мир.-1975.-541 с.
4. *Пискунов В. Г*., Вериженко В.Е. Линейные и нелинейные задачи расчета слоистых конструкций.-К.: Будівельник, 1986.-176 с.
5. *Бабенко А. Є*. Застосування і розвиток метода покоординатного спуску в задачах визначення напружено-деформованого стану при статичних та вібраційних навантаженнях .-К: КПИ,1996.-96 с.
6. *Altenbach, H*., Altenbach, J .,Naumenko, K. Ebene Flächentragwerke. 2004, S. 1–210. ISBN: 9783642058417. DOI: 10.1007/978-3-662-06431-3