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THE FORCED VIBRATIONS OF PLATES ON ELASTIC FOUNDATION CONSIDERING THE MATERIAL'S CREEP

Introduction

The most of the engineering structures for a long time are working under elevated temperature conditions and difficult loads. As a result, such conditions are leads to irreversible deformations in the structural elements.

In the aircraft industry there are a large number of plate-shell structures, which are working in conditions that are required to take into account the material's creep. Moreover, these structures are usually exposed to vibratory stress. Consequence of the influence of vibration can serve as a premature destruction of an individual components or as a whole unit. Therefore, even on the level of a small vibratory loads for structural materials that are subjected to the considerable stress during the operation to give rise to a limited lifespan, are additional load, which, as studies showed, it's necessary to take into account.

The problem's statement and solving

We're considering the forced oscillation plate on an elastic foundation, subject to material creep.

Equations of motion of the plates will look like:

$$\rho h \frac{\partial^2 w(x,y,t)}{\partial t^2} + D\nabla^4 w(x,y,t) - K_2 \nabla^2 w(x,y,t) + K_1 w(x,y,t) = q(x,y,t) + f;$$

$$D = \frac{Eh^2}{12}\left(1-\nu^2\right). \qquad (1)$$

Here ρ is the density of the plate material; h is the thickness of the plate; $w(x,y,z)$ is the deflection of the middle surface; K_1, K_2 are the coefficients of elastic foundation; ∇^4 is the biharmonic operator; D is the flexural rigidity; f is the elastic reaction of the base ;ν is the Poisson's ratio.

We're assume that a transverse load and the elastic reaction of the base are the lesser and the plate's material is described by creep's law [1] wherein the modulus of elasticity E is represented as a nonlinear operator

$$E = \left[E_1 + \varepsilon K \left(1 + \frac{E_1}{\varepsilon E_2}\right)\frac{\partial}{\partial t}\right] \Big/ \left(1 + \frac{K}{E_2}\right)\frac{\partial}{\partial t}. \qquad (2)$$

Where $E1$, $E2$, K are the constants of the plate's material.

By the substituting (2) into equation (1), we're obtain

$$\frac{\partial^3 w}{\partial t^3}+\xi\frac{\partial^2 w}{\partial t^2}+\rho h\Big(\left(D_1\nabla^4 w-K_2\nabla^2 w+K_1 w\right)\frac{\partial}{\partial t}+ +\xi\left[D_1\nabla^4 w-K_2\nabla^2 w+K_1 w\right]\Big)= =\rho h\varepsilon\left(f+\frac{\partial f}{\partial t}-\frac{E_2 D_1}{E_1}\nabla^4 w\frac{\partial}{\partial t}+\xi t+\frac{\partial q}{\partial t}\right). \tag{3}$$

Here're indicated $\xi=\frac{E_2}{K}$, $w=w(x,y,t)$,

For simplicity of calculations we are assume $\rho h=1$.

Apply to equation (3) Bubnov-Galerkin method:

$$w(x,y,t)=F_{r,s}(x,y)T_{r,s}(t) \tag{4}$$

where $F_{rs}(x,y)$ – is an eigenmode; r, s = 1, 2, …, N.

We're denote Q_{rs} the corresponding frequency, which is determined from the expression (3) with the boundary conditions type of Navier

$$F_{r,s}(xy)=\sin\frac{r\pi x}{b}\sin\frac{s\pi y}{c}. \tag{5}$$

$$Q_{2s}=D_1\left(\left(\frac{r\pi}{b}\right)^2+\left(\frac{s\pi}{c}\right)^2\right)^2+K_2\left(\left(\frac{r\pi}{b}\right)^2+\left(\frac{s\pi}{c}\right)^2\right)+K_1. \tag{6}$$

Here, b, c – are the lengths of sides of the plate.

Equation (3) up to a small parameter ε is satisfy the decision [2] under a harmonic force loading $q=q_0\sin\xi t$ in near-resonant region $Q_{11}^2=\xi^2+\varepsilon\Delta$ (Δ is the small detuning frequency).

$$w(x,y,z)=aF_{11}(x,y)\cos\Phi+\varepsilon U_1(x,y,a,\Phi,\Omega) \tag{7}$$

Where's $\Phi=\xi t+\psi$, and function $U_1(x,y,z,\Phi,\Omega)$ is a periodic function on Φ, Ω. Let's the right-hand side of equation (3) has the form:

$$p=\xi\left(-kw^3-k_2\frac{1}{2}\left(\left(\frac{\partial w}{\partial x}\right)^2\frac{\partial^2 w}{\partial x^2}+\left(\frac{\partial w}{\partial y}\right)^2\frac{\partial^2 w}{\partial y^2}\right)\right)+ +\left(-k_1w^3-k_2\frac{1}{2}\left(\left(\frac{\partial w}{\partial x}\right)^2\frac{\partial^2 w}{\partial x^2}+\left(\frac{\partial w}{\partial y}\right)^2\frac{\partial^2 w}{\partial y^2}\right)\frac{\partial}{\partial t}+ +\xi q_0\sin pt+q_0 p\cos pt-\frac{E_2D_1}{E_1}D^4 w\frac{\partial}{\partial t}+\xi q+\frac{\partial q}{\partial t}. \tag{8}$$

Then the amplitude a and phase of a Ψ are depend on time and are determined numerically from the first approximation the solution (7).

$$2p\frac{da}{dt} = -h\xi a - p_0\cos\psi_0$$

$$2pa\frac{d\psi}{dt} = Q_{11}^2 - p^2 - Qa^3 + \tilde{h}Q_{11}a + P_0\sin\psi_0. \qquad (9)$$

Here is

$$\tilde{h} = D_1E_2\left(\frac{\pi^2}{b^2}+\frac{\pi^2}{c^2}\right)\frac{\varepsilon}{E_1}\left(Q_{11}^2+\xi^2\right), \qquad P_0 = \frac{16q_0\varepsilon}{\pi^2},$$

$$Q = 9\left(-3k_1 + k_2\left(\frac{\pi^4}{b^4}+\frac{\pi^4}{c^4}\right)\frac{1}{2}\right)\frac{\varepsilon}{64}. \qquad (10)$$

In the case of steady oscillations the equation of amplitude-frequency curves are simplified and becomes

$$f\left(a_0, p^2\right) = \left(a_0\left(Q_{11}^2 - p^2\right) - Qa_0^3 + hQ_{11}^1a_0\right)^2 - p_0 + \tilde{h}\xi^2p^2a_0 = 0\ . \qquad (11)$$

To study the stability and stationary oscillations we substitute $a = a_0 + \delta a;\ \Psi = \Psi_0 + \delta\Psi$ into equations (9) and obtain

$$2p\frac{d\partial a}{dt} = -h\xi p\partial a + p_0\sin\psi_0\partial\psi,$$

$$2ap\frac{d\partial\psi}{dt} = \left(\left(Q_{11}^2 - p^2\right) - 3Qa_0^2 + \tilde{h}Q_{11}^2\right)\partial a + p_0\cos\psi_0\cdot\partial\psi. \qquad (12)$$

The characteristic equation for the system (12) has the following form:

$$4\lambda^2p^2 + \left(\left(Q_{11} - p^2\right) - Qa_0^2 + \tilde{h}Q_{11}^2\right)\left(Q_{11}^2 - p^2\right) - 3Qa_0^2 + hQ_{11}^2 + \tilde{h}^2\xi^2p^2 = 0 \qquad (13)$$

The stability condition for stationary oscillation has the form

$$\frac{\partial f\left(a_0, p^2\right)}{2a_0\partial a_0} > 0\,. \qquad (14)$$

Using the condition (14) we can determine the boundaries of stability oscillations. Thus, the expression (11) shows that the creep reduces the amplitude.

To investigate the stability of complex oscillation of plate-shell structures, which are widely used in the aircraft industry, power engineering and other industries the main problem is to determine the spectrum of the natural frequencies and mode shapes. In connection with the complexity of the geometric shape of objects to determine the natural frequencies and mode shapes is advisable to use numerical methods. One of efficient numerical methods for solving the problem of oscillations is a method for increasing the stiffness [3], [4], which is based on variational-grid approach of construct a functional type of Rayleigh and minimize its by method of coordinate wise descent [4], which is a stable iteration algorithm.

Conclusions

1. The forced vibrations of a rectangular plate with taking into account creep are investigated by using the asymptotic methods.
2. The stability of stationary oscillations was investigated.
3. The analytical dependences, which are showing the influence of creep on the amplitude of the forced oscillations are presented.
4. The recommendations in the investigation of vibrations of complex mechanical

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