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# ON THE THEORY OF RAYLEIGH WAVES CAUSED BY A COLLISION

**Ua** В роботі розглядається залежність характеристик хвиль Релея, викликаних ударом, від властивостей поверхонь тіл співударяються.

Основні характеристики процесу удару, - тривалість удару, місцеве стиснення, напруження в області контакту, - розглядаються як функції параметрів, що характеризують частоти, амплітуди, довжини хвиль Релея, та швидкість їх згасання.

В попередніх роботах авторів було розглянуто випадок співудару тіл, недеформовані поверхні яких мають велику кривину. В даній роботі досліджується випадок співудару тіл із малою кривиною їх недеформованих поверхонь.

Порівняння результатів, отриманих авторами у даній роботі, з результатами отриманими раніше дозволило зробити висновок : спектр частот хвиль Релея не залежить від геометричних властивостей поверхонь тіл, що співударяються. Вочевидь, суттєвий вплив кривина поверхонь тіл може мати на амплітуду хвиль та швидкість їх згасання з глибиною.

**Ru** В работе рассматривается зависимость характеристик волн Релея, вызванных ударом, от свойств поверхностей соударяющихся тел.

Основные характеристики процесса удара, - продолжительность удара, местное сжатие, напряжения в области контакта, - рассматриваются как функции параметров, характеризующих частоты, амплитуды, длины волн Релея и быстроту их затухания.

В предыдущих работах авторов был исследован случай соударения тел, недеформированные поверхности которых обладают большой кривизной. В данной работе рассмотрен случай соударения тел с малой кривизной их не-

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деформированных поверхностей.

Сравнение результатов, полученных авторами в данной работе, с результатами, полученными ранее, позволило сделать вывод: спектр частот волн Релея не зависит от свойств поверхностей соударяющихся тел. Очевидно, существенно влияет кривизна поверхностей тел на амплитуды волн и быстроту их затухания с глубиной.

## Introduction

About a hundred years ago, H. Hertz put forward the theory of contact static interaction of elastic bodies and the theory of collisions of such bodies as a secondary result of his research on electrodynamics. Both of these theories have been the subject of numerous studies to date.

In this paper, it is the theory of collisions of elastic bodies that is considered.

H. Hertz theory was diametrically opposed to the Saint-Venant theory of elastic body impacts. While Saint-Venant considered the dynamic state of colliding bodies as a whole, H. Hertz proposed to consider the deformation field in the small region adjacent to the surface of contact dynamic compression. Outside this area, H. Hertz proposed to neglect deformations, i. e. to consider these body parts as absolutely solid.

H. Hertz assumed that the impact of elastic bodies leads to the appearance of various oscillatory movements of their elements. These unsteady oscillatory movements, spreading in the body, over time can take the form of stationary oscillations, which, in particular, belong to the surface Relay waves, Love waves and other types.

The analysis of dynamic contact interaction of elastic bodies in the presence of the influence of surface waves confirms the assumption made by H. Hertz that surface waves arising from the impact of elastic bodies significantly affect the impact process and the phenomena accompanying it [1].

The papers [1 – 3] present the results of the study of the appearance of Relay waves in the collision of elastic bodies and consider the main characteristics of this dynamic process.

In papers [5 – 7] the main characteristics of the impact process – the recovery factor, local compression, duration of the impact, stress in the body in the contact area, etc. – were determined depending on the main parameters characterizing the surface waves that occurred during the impact. Such parameters are amplitudes of expansion and shift waves, frequency of these waves, and their attenuation with depth.

The force of dynamic interaction $P$ and local compression $\alpha$ were defined as a function of two parameters: frequency $\omega$ and the parameter $\varepsilon$, which determines the dependence of the characteristics of the Relay waves on the geometric properties of the colliding surfaces. The solution to the problem was given in [3] for the case ε <1, which corresponds to the collision of bodies whose non-deformed surfaces have high curvature.

**Problem Statement**

The purpose of this paper is to investigate the impact of bodies whose un-deformed surfaces have low curvature, i. e. ε > 1.

A direct central impact of two rotation bodies is considered. The initial contact of bodies is carried out at one point. At this point of contact of the bodies, before the start of the collision process, the beginning of the cylindrical coordinate system is selected. The axes $OZ_1$ and $OZ_2$ are drawn in the directions of the internal normals to the surfaces of undeformed bodies at the point of their contact.

Analytical formulae of displacements that determine the motion of elements of an elastic body during the propagation of surface waves have the following form [2]:

$$U_r = -(M æ e^{(-\xi z)} + N\eta e^{\mu} z)\cos\omega t I_1(æ r),$$
$$U_z = -\left(M\xi e^{-\xi z} + N æ e^{-\eta z}\right)\cos\omega t I_0\left(æ r\right) \tag{1}$$

where $M$, $N$, ω, æ, ξ, η – mean amplitude and frequency characteristics of a dynamic process.

The conditions of the absence of additional strains on the surfaces of bodies outside the contact point and inside it result in the relations between the unknown parameters in (1):

$$2M æ\xi + N(\eta^2 + æ^2) = 0;$$
$$M\xi^2 - \lambda æ^2 + 2\mu æ\eta N = 0; \tag{2}$$
$$\xi^2 = æ 2 - \frac{\omega^2}{C_1^2}; \quad \eta^2 = æ^2 - \frac{\omega^2}{C_2^2};$$

where $C_1$ and $C_2$ are the propagation velocity of expansion waves and distortion waves in an elastic body.

The basic equations of the dynamic contact problem have the following form [1]:

$$\int_0^{2\pi}\int_0^{1} p\left(r,t\right) r dr d\varphi = \frac{-m_1 m_2}{m_1 + m_2}\alpha';$$
$$\left(\upsilon_1 + \upsilon_2\right)\int_0^{2\pi}\int_0^{\rho} \frac{p\left(r,t\right) r dr d\varphi}{\sqrt{r^2 - 2rr'\cos\varphi + r'^2}} = \alpha - A\left(1 + \varepsilon\cos\omega t\right) r^2, \tag{3}$$

where $p(r,t)$ is the contact pressure; $m_1, m_2$ are masses of colliding bodies; $\upsilon_1; \upsilon_2$ are elastic constants; $\alpha$ is local compression; $\varepsilon = \frac{æ^2}{4A}(M\xi + Næ)$ is the main parameter determining the presence of surface waves.

**Problem solution**

Papers [1, 3] contain a solution to this problem for the case $\varepsilon < 1$, which corresponds to the collision of bodies whose undeformed surfaces have a substantial curvature.

The aim of this paper is to study the case when $\varepsilon_1$.

Let's decompose the original function $P$* [3] by degrees of the parameter $\delta = 1/\varepsilon\delta$ = 1/ε and make the necessary transformations. We get the force of dynamic interaction of $P(t)$ in the form of decomposition by degrees δ:

$$\begin{aligned}
P(t) = \frac{1}{\sqrt{\pi}}\left(\frac{\upsilon_0}{k}\right)^{\frac{3}{2}} \Bigg\{ & \delta^{\frac{1}{2}}\left[\frac{2^2}{1\cdot 3}t^{\frac{3}{2}} + \frac{1}{2}\omega^2\frac{2^4}{1\cdot 3\cdot 5\cdot 7}t^{\frac{7}{2}} - \frac{1}{2\cdot 4}\omega^4\frac{2^6}{1\cdot 3\cdot 5\cdots 11}t^{\frac{11}{2}} + \cdots\right] - \\
& -\delta\frac{3}{2}\frac{\sqrt{\pi\upsilon_0}}{mk^{\frac{3}{2}}}\left(\frac{t^4}{4!} + \omega^2\frac{t^6}{6!} + \cdots\right) - \delta^{\frac{3}{2}}\left[\frac{1}{2}\left(\frac{2^2}{1\cdot 3}t^{\frac{3}{2}} + \frac{3}{2}\omega^2\frac{2^4}{1\cdot 3\cdot 5\cdot 7}t^{\frac{7}{2}} + \cdots\right) - \right. \\
& \left. -\frac{21}{8}\frac{\upsilon_0\sqrt{\pi}}{m^2k^3}\left(\frac{2^7}{1\cdot 3\cdot 5\cdots 13}t^{\frac{13}{2}} + \frac{3}{2}\omega^2\frac{2^9}{1\cdot 3\cdot 5\cdots 17}t^{\frac{17}{2}} + \cdots\right) + \right. \\
& +\delta^2\left[\frac{3}{2}\frac{\sqrt{\pi\upsilon_0}}{mk^{\frac{3}{2}}}\left(\frac{t^4}{4!} + 2\omega^2\frac{t^6}{6!} + \cdots\right) - \frac{5\sqrt{\pi}\upsilon_0^{\frac{3}{2}}}{m^3k^{\frac{9}{2}}}\left(\frac{t^9}{9!} + 2\omega^2\frac{t^{11}}{11!} + \cdots\right)\right] + \\
& +\delta^{\frac{5}{2}}\left[\frac{1\cdot 3}{2\cdot 4}\left(\frac{2^2}{1\cdot 3}t^{\frac{5}{2}} + \frac{5}{2}\omega^3\frac{2^4}{1\cdot 3\cdot 5\cdot 7}t^{\frac{7}{2}} + \cdots\right) - \right. \\
& \left. -\frac{3}{2}\frac{21}{8}\frac{\upsilon_0}{m^2k^3}\left(\frac{2^7}{1\cdot 3\cdot 5\cdots 13}t^{\frac{13}{2}} + \frac{5}{2}\omega^2\frac{2^9}{1\cdot 3\cdots 17}t^{\frac{17}{2}} + \cdots\right)\right] \\
& -\delta^3\left[\frac{3}{2}\frac{\sqrt{\pi\upsilon_0}}{mk^{\frac{3}{2}}}\left(\frac{t^4}{4!} + 3\omega^2\frac{t^6}{6!} + \cdots\right) - \frac{5}{2}\frac{\sqrt{\pi}\upsilon_0^{\frac{3}{2}}}{m^3k^{\frac{9}{2}}}\left(\frac{t^9}{9!} + 3\frac{t^{11}}{11!}\omega^2 + \cdots\right)\right] - \ldots \Bigg\}.
\end{aligned} \tag{4}$$

To find the parameters δ and ω, we use the function that determines the enforcement of the system [4]. In this case

$$Z = (\delta + \cos\omega t)^{\frac{2}{3}}\left\{\frac{4}{3}t^{\frac{3}{2}}\left(1 - \frac{\delta}{2}\right) + \frac{8}{105}\omega^2 t^{\frac{7}{2}}\left(1 + \frac{3}{2}\delta\right)\right\}^{\frac{4}{3}}. \tag{5}$$

As previously [1], we require that at each point of the interval $T(\varepsilon,\ \omega)$, equal to the duration of the impact, the function is least deviating from zero [4]. The duration of the strike in this case is equal to

$$T = T_0 \frac{1+\frac{1}{5}\delta}{\delta^{\frac{1}{5}}},$$

where $T_0$ is impact duration without taking into account the influence of surface waves [1].

Performing a coordinate transformation so that

$$t = \frac{\tau + \frac{T_0}{2}}{\delta^{\frac{1}{5}}},$$

we will bring this problem to the problem of Chebyshev [4]. According to the Chebyshev theorem, we obtain the equations:

$$\frac{3}{5}\left(\delta+\frac{7}{6}+\frac{2}{15}\cdot\frac{\psi^2}{\delta}+\frac{3}{35}\psi^2\right)\sin\psi-\frac{4}{35}\psi(2-3\delta)(1+\frac{1}{5}\cdot\frac{\psi}{\delta}\sin\psi)=0;$$
$$\frac{4}{35}\psi^2(2-3\delta)\sin\psi-\left(1-\frac{\delta}{2}+\frac{2}{15}\psi^2-\frac{1}{5}\delta\psi\right)\sin\psi=0. \quad (6)$$

Here $\psi=\omega\left(\tau+\frac{T_0}{2}\right)\delta^{\frac{1}{5}}$.

As a result of studying and solving the system (6) and the boundary conditions for the function $Z$, we determine the spectrum of $\delta$ and $\omega$. We find that the frequency spectrum of Rayleigh waves is the same as for the case $\varepsilon<1$.

The lowest frequency is: $\omega_1 \cong 2{,}8\upsilon_0^{\frac{1}{5}} \Big/ m^{\frac{2}{5}}k^{\frac{3}{5}}$.

## Summary

A comparison of these results with those obtained previously [3] allows us to conclude that upon the collision of bodies whose surfaces have small curvature, i. e. $\varepsilon<1$, the frequency spectrum is the same as for the case $\varepsilon<1$. The curvature of the surfaces of bodies significantly affects the amplitudes of Rayleigh waves and their attenuation rate with depth. This issue needs further research.

## References

1. *Kilchevsky N. A.* Dynamic contact compression of elastic bodies. Collision. K., Naukova Dumka, 1976.

2. *Lyav A.*, The mathematical theory of elasticity. M., ONTI, 1935.
3. *Kilchevsky N. A., Ilchishina D. I.*, On the surface waves arising from the collision of elastic bodies. PM, v.5, i.7, 1969.
4. *Chebyshev P. L.*, Issue of the smallest quantities. Coll.works, Vol. 2, M.-L., 1947.
5. *Ivanova O. N., Ilchyshina D. I.* Influence of Rayleigh waves on the impact process. MGS, No. 28, 2014.
6. *Ivanova O. N., Ilchyshina D. I.* Approximate methods for estimating the coefficient of recovery in coexistence of elastic bodies. MGS, No. 29, 2015.
7. *Ivanova O. N., Ilchyshyna D. I., Chernenko S.*, The extended theory of elastic bodies collision. ICMK, №19, 2018.